

Instructions: This is a 50 minute test similar in style to the three hour final exam. Attempt all questions. Marks total 45.

1. (18=6+6+6 marks) Write a paragraph, in your own words, on each of the following topics. You will be marked on the extent to which you reveal that you understand the statement.

a) The jargon term "Statistical Significance" is quite different from the everyday meaning of "Significance".

a) Statistical Significance refers to a hypothesis testing situation: when the null hypothesis is rejected, the test is said to be "statistically significant". However, the everyday "significance" depends on whether the result of the test is important to the person asking the question about the hypothesis. A hypothesis test might have important implications whether it turns out to be statistically significant, or not statistically significant.

b) The Poisson Process involves the Poisson distribution, but is closely related to the exponential and gamma distributions as well.

b) The Poisson Process is a counting process $N(t)$ which counts the number of events during $(0,t)$. $N(t)$ itself has a Poisson distribution. The time between events has an exponential distribution. The time between k events has a Gamma distribution with shape parameter k . (Give a bonus mark if the student explains that the rate constant λ in the exponential distribution is related to the mean λt of the Poisson, and the scale constant β of the Gamma = $(1/\lambda)$.)

c) A 95% Confidence Interval is **not** expected to include about 95% of the sample observations on which it is based, in a typical sampling scenario.

c) The width of a 95% Confidence interval is determined by a multiple of the SD of the statistic on which it is based, and this multiple usually decreases with increasing sample size. In other words the CI narrows as the sample size increases, whereas the SD of the distribution of sample observations will not trend up or down as the sample size increases. (Bonus mark if this general explanation is given (with reference to "statistic") – but full marks (6) if this is described in terms of the sampling distribution of the sample mean.)

2. (10 =5+5 marks) a) A population consists of 2 red beads and 3 white beads. A random sample of size 10 is selected with replacement. If R is the proportion of red beads among the 10 sampled beads, what is the probability that R is greater than 0.6.

a) population proportion p is $2/5 = 0.4$. $n=10$ in sample. $P(R>.6) = P(\text{no. of red beads} \geq 7) = \text{binomial probability from Table A.1 (or calculator)} = 1 - .945 = .055$. (take off one mark if only mistake is ≥ 6 instead of ≥ 7 , so they get 0.166 instead).

b) Find an approximate value for the probability in a) if the sample size is 100.

b) $P(R>.6) = P(61 \text{ or more red beads})$: use normal approx. to binomial $z = (60.5 - 40) / (.4 * .6 * 100)^{1/2} = 4.1$ and $P(Z > 4.1) = 0$. Note: you can also work with the proportions: $.6 - .4 / (.4 * .6 / 100)^{1/2} = 4.1$... (Since the 60.5 gives the same answer as 60 or 61, I did not deduct one mark for 60 or 61.)

3. (10 = 8+2 marks) A student project involves selecting a random sample of households in a certain city and determining if anyone is home at noon. Student A selects 100 households and finds 50 of them have someone home at noon. Student B selects 150 households in a different city and finds 90 of them with someone home.

a) Find a 95% confidence interval for the difference of proportions of home-at-noon between the two cities.

b) Do you think there is a non-zero difference between the two population means? Say why.

3. a) difference of proportions is $.5 - .6 = -.1$. SD of difference is $(.5 * .5 / 100 + .6 * .4 / 150)^{1/2} = 0.064$ so the 95% CI for the difference of proportions is $-0.1 \pm .128$

b) No. CI includes 0 so 0 is credible value of difference.

4. (7 marks) A deck of 52 playing cards has 4 suits and 13 denominations in each suit. In a poker hand of 5 cards dealt at random from a shuffled deck, what is the probability of getting 4 of the same denomination in the 5-card hand? (You must record enough detail to reveal how you compute this probability, as well as work out the numerical answer.)

4. There are 13 denominations, and the number of hands containing a particular one of them is 48, since there are 48 ways to make up the fifth card. So the probability is $13 * 48 / C_{52,5} = .00024$.

(KLW 070330)