Stat 280 Oct. 22, 2001

Where are we? Ch 6: Special Continuous RVs.

Uniform pdf=1/(b-a) on (a,b) cdf = x/(b-a) mean=(a+b)/2 SD= $\frac{|b-a|}{\sqrt{12}}$

Expo pdf= $\lambda e^{-\lambda x}$ on x>0 cdf = 1- $e^{-\lambda x}$ mean= λ^{-1} = SD

Normal pdf = (see p 241) cdf by table or algorithm only, mean= μ SD= σ

Need to know graphs of each and dependence on parameters

Also,
Application examples:

Uniform: last digit of race timer, direction of bird flight, random decimal no.s
Expo: interarrival time, survival time
Normal: averages, totals, meaasurements subject to many uncontrolled errors, Approx to Binomial and Poisson (via mean and SD).

Recall Poisson Y mean m and SD $\sqrt{m}$, so P(Y<y) approx =
P[Z = (Y-m)/ $\sqrt{m}$ < y-m/($\sqrt{m}$)] where Z is N(0,1)

Binomial Y mean np and SD = $\sqrt{np(1-p)}$ so P(Y<y) approx =
P[Z=(Y-np)/ $\sqrt{np(1-p)}$ < (y-np)/ $\sqrt{np(1-p)}$] where Z is N(0,1).

Any probability distribution can be approximated by the N distribution. But the Poisson and Binomial are well approximated if:

Poisson: m is large
Bin: n large (as for CLT) (Binomial is sum of IID Bernoullis)

6.3 Gamma pdf p 251, cdf available only as table or algorithm, mean= $\alpha\lambda^{-1}$ and SD= $\alpha^{1/2}\lambda^{-1}$

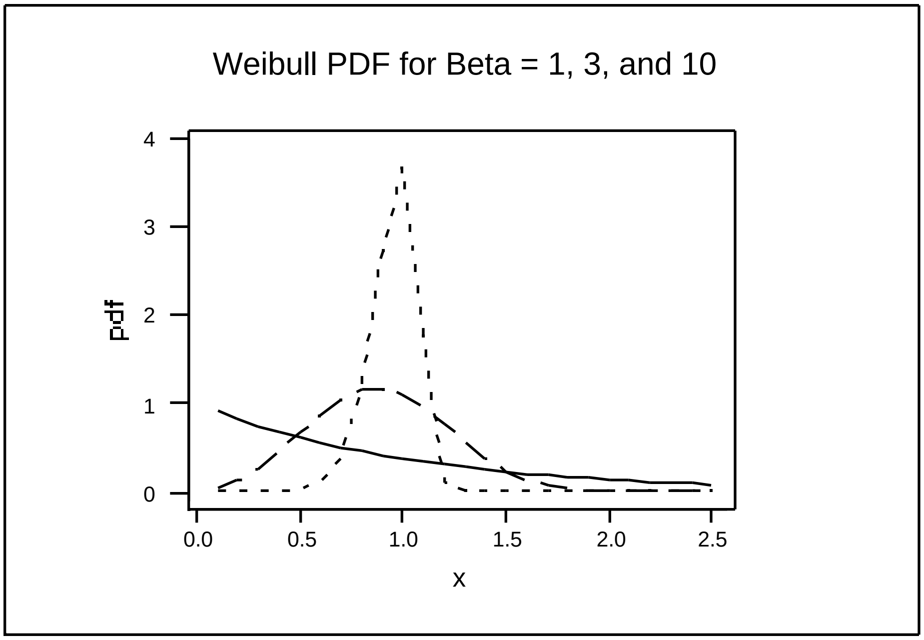
An example of Gamma, Y = sum of α exponentials, waiting time to α^{th} event.

Recall CLT suggests Gamma approaches Normal as α increases.

But for $\alpha=1$, Gamma is Exponential.

Note: text uses β for scale parameter and $\beta=\lambda^{-1}$

Weibull: pdf p 255 cdf like exponential but with extra exponent See p 255.
Mean SD p 256
Warning: Note in Weibull, scale parameter is now α



Expo is special case of Weibull, and is special case of Gamma.

Weibull is usually used for survival times (or lifetimes) when the constant failure rate assumption does not hold. Weibull Failure rates can be increasing with time, or decreasing with time.

Failure rate h(t) = f(t)/[1-F(t)] in general for continuous lifetime distribution and for Expo h(t)= λ. Check this. (or see p 382 and section 10.2)
Weibull h(t) = $\beta\lambda^{-1}(t\lambda^{-1})^{\beta-1}$and is increasing (decreasing) in t depending if $\beta>1$ (<1)

6.5 Moment Generating Functions

Recal $M_X(t) = E(e^{tx})$

Uses of MGF:

1. Compute moments by differentiation
2. Identify CDF from MGF
3. Working with sums of IID Rvs

(The lecture ended about here ….)

$M_{X+Y}(t) = M_X(t)*M_Y(t)$

Check MGF of Gamma cf Expo.

6.6 Methods of Moments Estimation

Method for determining estimator formula for a parameter when the pdf involving the parameter is known:

Example: Uniform $(0,\theta)$ Data $X_1, X_2, \ldots, X_n$

How to estimate θ?

$E(X) = \theta/2$ is known. But sample mean estimates $E(X)$.

So est of θ is 2* sample mean.

See example 6.6-2 for less trivial example.