

Today: More Ch 7 (Sections 7.4 and part of 7.5)

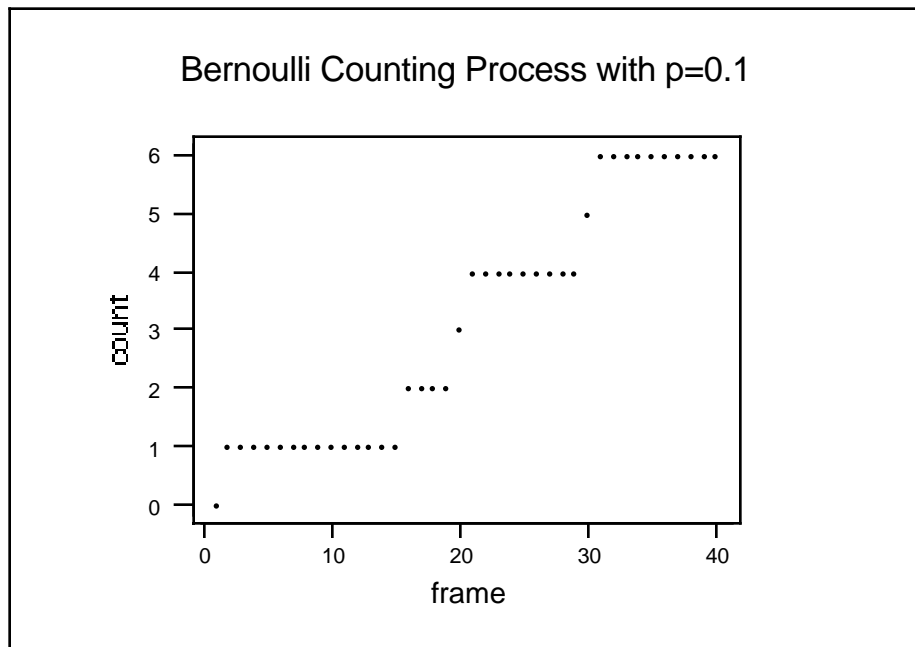
Midterm will cover Ch 7 to section 7.4

Review session will be Nov 5.

Exercises to try (answers in book): 7.1-5, 7.2-3, 7.3-3, 7.4-7

Where were we last Wednesday?

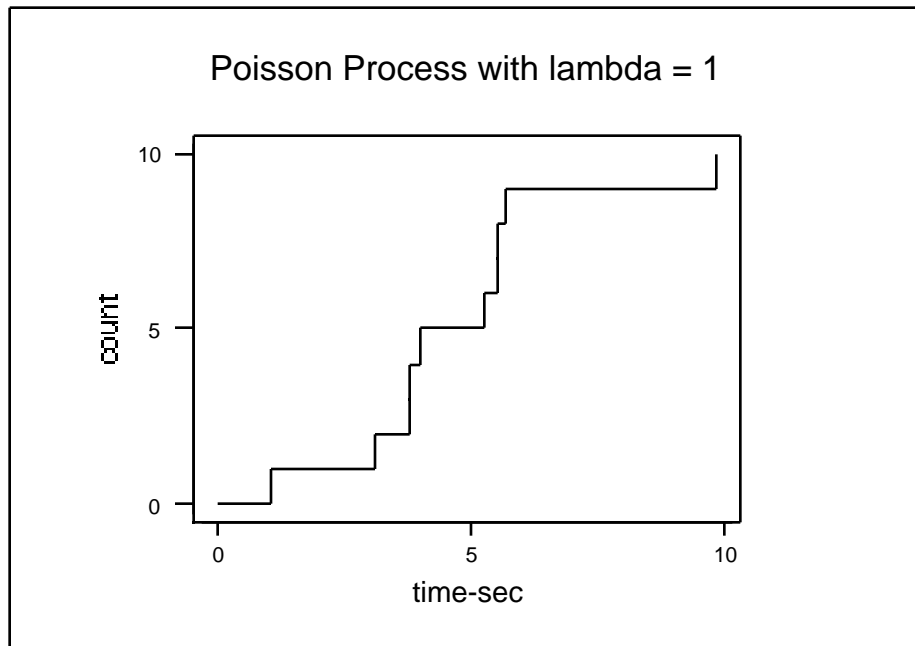
Sec 7.1: Bernoulli Counting Process: modeled  $X(t)$ , the number of events occurring in time  $1, 2, 3, \dots, t$



Example: frame, 1 minute

Number of accidents in a large city, 8:00 am to 8:40 am.

Sec 7.2: The Poisson Process: modeled  $N(t)$ , the number of events occurring in time  $(0,t)$ .



In both cases, we assume the average rate of arrival,  $\lambda$ , is constant, but the rate of arrival will of course vary in particular outcomes. (So  $X(t)$  and  $N(t)$  are random variables.)

Ex 7.1-4 Two researchers A and B use different frames to model customer arrivals at a grocery store, during a one hour period.

A uses frames of 1 second

B uses frames of 1 minute

Customers arrive at the store at the average rate of 36 customers per hour

a) For a single frame, what is  $P(\text{an arrival})$ ?

$P(\text{arrival in a frame}) = 36/3600 = .01$  for A  
 $= 36/60 = 0.6$  for B

Reasonable because B's frames are 60 times larger.

b) Number of customers arriving in an hour? Mean and SD?

Mean is still 36 for one hour in both cases. Number of frames \*  $P(\text{arrival}) = 36$

Var is  $np(1-p) = 3600 * .01 * .99 = 35.64$  for A and  $= 60 * .6 * .4 = 14.4$  for B

SD is 5.9 for A

3.8 for B

c) How would the two simulations compare? B's would be too stable, and not reflect the true variability of the arrival process.

Q: What would comparable SD of Poisson model?

Mean for 1 hour is 36.  
 SD of Poisson(36) is 6.0  
 A's simulation was close.

Exercise 7.2-4

Artic equipment.

One service every 200 hours, on average. Rate is .005 services per hour.

a) Mission takes 24 hours. P(no service) = ?

Mean number of services for 24 hours is  $24 * .005 = .12$ , so

$P(\text{no service in 24 hours}) = e^{-0.12} = 0.89$  is the probability that the mission can be completed without a service break.

b) If  $e^{-t} = 0.95$  with  $\lambda = .005$  then  $-.005t = \ln(.95)$   $t = 10.26$  hours

The mission would have to be completed in 10.26 hours.

Section 7.4 The Bernoulli Single server Queuing Process

Arrivals Process: In each frame,  $P(\text{arrival}) = P_A$

Service Process: In each frame,  $P(\text{departure} | \text{at least one in queue}) = P_S$

Easy to simulate with two Bernoulli processes running in the same time-frames.

Note Markov transitions from one frame to next, for queue length.

0->0: conditional probability of such a transition =  $1 - P_A$

0->1:  $= P_A$

$i \rightarrow i-1$  when  $i \geq 1$ :  $= P_S(1 - P_A)$

$i \rightarrow i$  when  $i \geq 1$ :  $= (1 - P_A)(1 - P_S) + P_A P_S$

$i \rightarrow i+1$  when  $i \geq 1$ :  $= P_A(1 - P_S)$

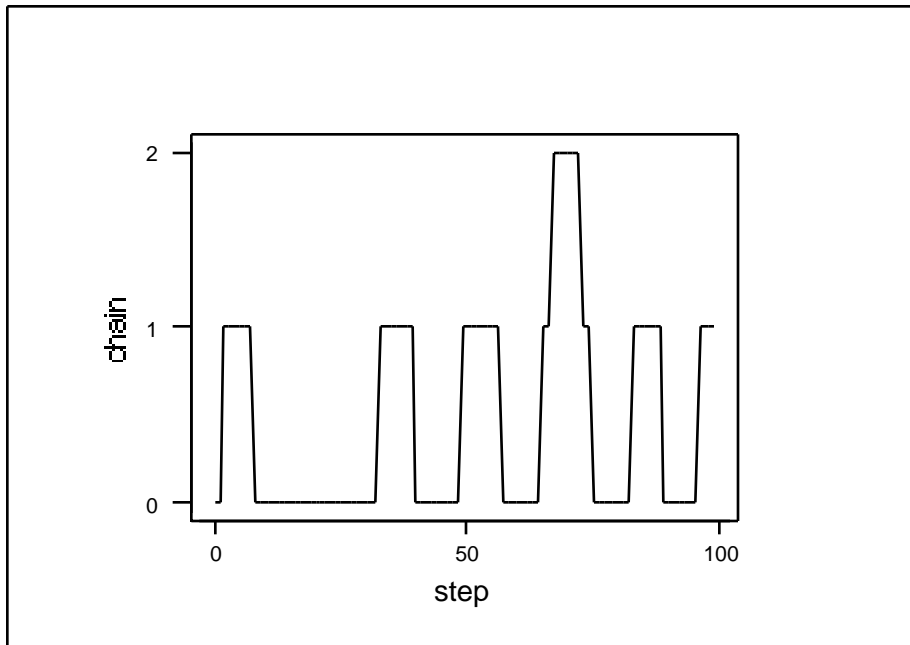
Could simulate this queue using the Markov Chain simulator.

Do we need an infinite dimensional 1-step transition matrix?

Use max queue = 6 for this one:

Matrix M1

0.900	0.100	0.000	0.000	0.000	0.000
0.135	0.780	0.085	0.000	0.000	0.000
0.000	0.135	0.780	0.085	0.000	0.000
0.000	0.000	0.135	0.780	0.085	0.000
0.000	0.000	0.000	0.135	0.780	0.085
0.000	0.000	0.000	0.000	0.135	0.865



We were OK assuming queue length  $<6$ .

### Section 7.5 The M/M/1 Queue

This section is about some queuing processes in continuous time. Of course the queue length that we model is still discrete: 0,1,2, ....

“F/G/s” notation for queues:

F is cdf of interarrival time

G is CDF of service time

s is number of servers

“M” is used to denote the exponential CDF because of its Memoryless property.

M/M/1 means exponential interarrival times, exponential service times, and 1 server.

The arrival rate is denoted  $\lambda$  and the service rate  $\mu$ .

Note that the service rate  $\mu$  is greater than the rate at which customers are served since if there is nobody in the queue, the service rate is 0 until a customer arrives.

We can use the Bernoulli Queuing Process as a starting point for defining a continuous time Queuing Process. Just let the frame size  $\rightarrow 0$ . (see p 292 for details) to get the M/M/1 queuing process.

Note how the independence of Bernoulli trials implies the memoryless property of the exponential interarrival times, assuming the limiting process works as claimed.

Preparation for next time: Read the important development of the steady-state distributions for the M/M/1 queue. The idea in this section applies more generally to queuing processes. Note also how important it is that in a queue, the transitions out of state  $i$  are limited to either  $i+1$  or  $i-1$ .

Appendix: Here is the program that generated the Markov Queuing Process in Example 7.4-1 (p 285).

Gmacro

MChain

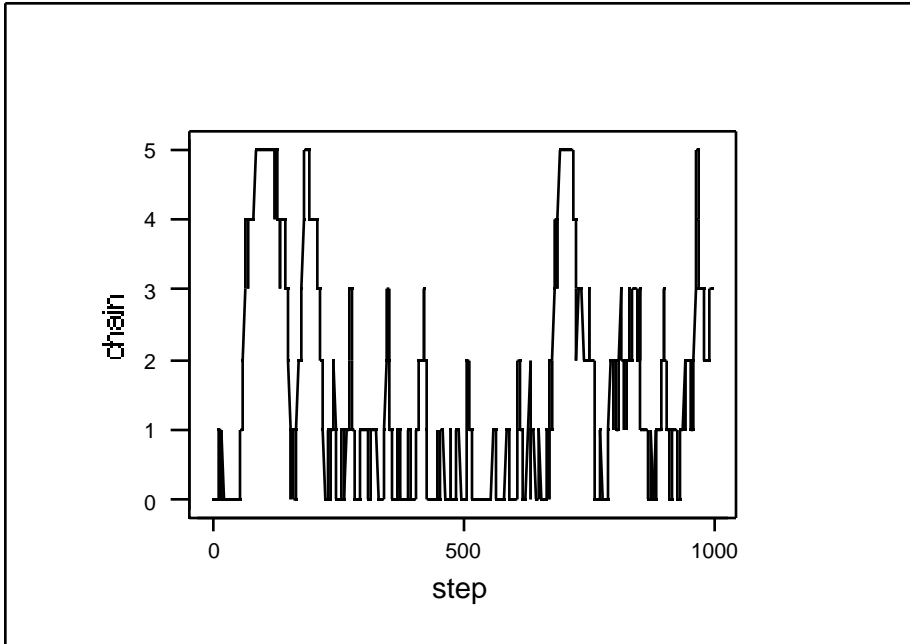
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# Here is a program that will simulate a Markov Chain: To adapt
# it to any Markov chain, just specify the size of the state space, k1,
# the transition matrix, c1-ck1, the number of time points to
# simulate, k4, and the start state, k2. The program produces a plot
# of the k4 values of the chain, and also an estimate of the steady
# state distribution.
#
# To use this program, Type %MChain after the MTB> prompt.
#
# We begin with the initialization
#
let k4=1000 # length of chain to be simulated
let k2=0 # set start state for Markov chain
let k1=6 # set size of state space of Markov chain
read c1-ck1 # read in k1 by k1 transition matrix
.90 .10 0 0 0 0
.135 .780 .085 0 0 0
0 .135 .780 .085 0 0
0 0 .135 .780 .085 0
0 0 0 .135 .780 .085
0 0 0 0 .135 .865
end
copy c1-ck1 m1 # we want the columns to be the distributions
trans m1 m2 # to generate so have to transpose transition
copy m2 c1-ck1 # matrix (See rand & discrete below)
let k6=k2+1 # we will want to change this "from" state at each loop
Let k1=k1-1 # We want our states to be 0,1,2,...,k1-1
set c100 # the column that makes up part of the table used
# to define the discrete probability distribution
```

```

O:k1
end
Let k4=k4-1 # We already have one of the k4 values from k2
set c200 # this stores resulting chain
# We just fill it with any numbers for now
1:k4
end
name c200 'chain'
do k5=1:k4
  rand 1 c101; # This uses the ck6 probabilities to generate a value
  discrete c100 ck6. # c100 is just 0,1,2,3,...,m-1 for m state MC
  let c200(k5)=c101(1) # capture generated state
  let k6=c101(1)+1 # record result for input to next loop
# Note that c1 holds probs for transition from state 0, c2 from state 1 ...
enddo
set c201 # this sets up the time variable in plot
O:k4
end
name c201 'step'
stack k2 c200 c200 # puts start value at beginning of chain
print c201 c200
table c200; # this just computes the proportion of time spent in
# each state. Compare with the steady state
# probabilities
colp. # column percents subcommand to table
plot c200*c201; # Plot of Markov Chain
connect. # joins the points by lines
# To get the output, just copy and paste from the MINITAB
# Session window to a word processor, OR, select a portion of the
# Session window and under the File menu select "Print Selection".
# The graph can be printed either by COPY GRAPH in the EDIT menu, and
# then paste to a word processor, OR by "Print Window" under the
# File menu - but for either of these, one has to find the graph at
# the bottom of the WINDOW menu.
Endmacro

```

Here is the graph of a longer simulation ....



Maybe a 6 by 6 transition matrix was not big enough after all!