STAT 280 Nov. 5, 2001

Review session:

Important Ideas and Techniques:

Ch 1: Basics
Counting techniques section 1.4 nCr and nPr formulas.
Law of Total Probability. Use of conditioning to simplify calculations. p 31.
Independence – product rule. p 35

Ch 2: Discrete Probability Models
Pmfs
Joint Distributions. Conditional, Joint and Marginal probabilities. P 53
Expected Value. Population mean. Exp Value of function of a rv. Sec 2.3
Variance and SD. P 65 of a+bX p 68
Simulation Sec 2.5
Var of aX+bY. Var of a mean. P 86
Exp Value of a conditioned RV. (a conditional prob dist is a dist). P 95

Ch 3: Discrete Models:

Discrete Uniform: P(X=x) = 1/n where x=1,2,…,n
Bernoulli – 0-1 P(X=1)=p
Binomial P(X=x) = nCx $p^x(1-p)^{n-x}$ x=0,1,…,n
Geometric P(X=x) = $(1-p)^{x-1}p$ x=1,2,3,…
Negative Binomial P(X=x) = (x-1)C(r-1) $p^r(1-p)^{x-r}$ x=r, r+1, r+2, …
Hypergeometric P(X=x) = rCx (N-r)C(n-x) / NCn x=0,1,2,…,n unless
restricted by n-(N-r) below or r above
Multinomial $P(Y_1=y_1, Y_2=y_2,\ldots, Y_k=y_k) = [n!/(y_1!\, y_2! \ldots y_k!)]\, p_1^{y_1} p_2^{y_2} \ldots p_k^{y_k}$
Where the $y_i$ are nonnegative integers and sum to n and $p_i$ sum to 1.
Poisson P(X=x) = $e^{-m} m^x / x!$ for x=0,1,2,…
Mean and SD for all these models.
Context for all these models.
Relationships between these models.
Moment Generating Functions: $M_X(t) = E(e^{tx})$
$M_{X+Y}(t) = M_X(t)\, M_Y(t)$
The moment generating function $M_X(t)$ identifies the rv X.
Ch 5 Continuous Random Variables
pdfs and cdfs – why not pmfs? Meaning of “density”
Probabilities from pdfs (integration)
Expected Value (population mean – integration formula) p 201
Probabilities and densities for a transformed RV p 206
Simulation using the fact that F(X) is U(0,1).
Joint, Marginal, and Conditional Densities. P 224.

Ch 6: Special Continuous RVs

Continuous Uniform: U(a,b) mean (a+b)/2 SD = $\frac{|b-a|}{2\sqrt{3}}$

Exponential $\lambda e^{-\lambda x}$ x>0 mean= $\lambda^{-1}$=SD models durations with constant hazard rate

Lack-of-memory property p 239

Normal models totals and averages. p 241

Normal Approximation of any dist using mean and SD.

Normal dist of aX+bY+c when X, Y normal.

Gamma f(x) = $\frac{x^{\alpha-1}e^{-x/\beta}}{\beta^{\alpha}\Gamma(\alpha)}$ x>0 models sum of α exponentials

Mean is $\alpha\lambda^{-1} = \alpha\beta$ SD is $\alpha^{1/2}\beta$

Weibull models durations with non-constant hazard. Details p 255-256

Stochastic Processes X(t) t discrete or continuous:

Ch 4: Markov Chains

"Simple" Queuing System (discrete time) uses Bernoullis and frames.

Is an example of a Markov Chain. (Why?)

Markov Property: Future given past and present only depends on present.

Markov Chain: X(0), State space {0,1,2,3,…} one-step transition matrix, P.

P has elements $p_{ij}$ = P(i->j) rows of P indexed by i, cols by j

Know how to read P(i->j) from row i and column j.

k-step transition matrix is $P^k$

long run (i.e steady state) probabilities from $P^k$ for large k.

or from Theorem 4.6-3 p 171

Simulation of a Markov Chain (How? P 161) (Why? P 162)

Return times, mean p 173.

First Passage Times and Absorbing States: sec 4.7

Ch 7: Markov Counting and Queuing Processes

7.1 Discrete time Markov Counting and Queuing Porcesses

Both kinds of process use Bernoulli RVs in a sequence of frames.
Sequence of independent Bernoulli trials.
Bernoulli Counting process: Geometric inter-event times, $T_1$, $T_2$, …
Count of events in frame t (t an integer here) is N(t) not quite Poisson.
Can model as a Markov Process. Need to relate rate of event occurrence to probability of an event in a frame.

7.2 Poisson Process (continuous time counting process).
Exponential inter-event times, $T_1$, $T_2$, …, and N(t) is Poisson.
Rate of events per unit time is λ. Time interval for count is $t_2$-$t_1$ or,

simply t if the time interval is (0,t). The mean of the Poisson RV
N(t) is $m=\lambda t$

7.3 Exponential interarrival times imply Poisson counts and vice-versa.

7.4 Bernoulli Queuing Process with single server.
Arrivals cause queue length N(t) to increase by 1, and service completion causes a drop by 1. Arrival times and service times imply the N(t) path and vice versa. Arrival rates and service rates must be converted to bernoulli probabilities (per frame) to apply this model.

The Bernoulli counting process is a Markov Chain.