STAT 280 Nov 14, 2001

More exercises from Ch 7 to try:

7.5-5, 7.6-1, 7.7-1, 7.8-1 have answers in the book.
7.5-1, 7.6-3, 7.9-1 do not have answers in the book but you can ask in tutorial if you are in doubt.

Assignment: Due Nov 28: 7.8-4, 7.9-4 (and include program in your submission).

Assignment due Nov 21 (Simulation Project)

Please send me an outline of your idea. I will reply if I think I can help you. Otherwise, proceed.

Today: More on 7.5-7.9 Continuous time queuing processes.

Continuous time, M/M/1, parameters arrival and service rates, $r=\lambda_A/\lambda_S<1$ (for steady state to exist).

Steady-state distribution of queue length explicit:

$$\pi_i = (1-\lambda_A/\lambda_S)(\lambda_A/\lambda_S)^i \quad \text{for } i = 0,1,2, \ldots.$$

e.g. taxi stand: arrival rate 3 per hour, service time averages 10 minutes. What are mean and SD of queue length?

Service rate is 6 per hour. Arrival service ratio is 3/6=1/2

So $\pi_i = (1-\lambda_A/\lambda_S)(\lambda_A/\lambda_S)^i = (1/2)^{i+1}$ for i = 0,1,2,…

So $\pi_0 = 1/2$, $\pi_1 = 1/4$, $\pi_2 = 1/8$, $\pi_3 = 1/16$,…

Mean queue length = 0*(1/2) + 1*(1/4) + 2*(1/8) + …
= 0 + .25 + .25 + .18 + .13 + .08 + .05 + .03 + .02 + .01 + 0
= 1.0

or use formula p 297 = r/(1-r) = 1
Similarly,
SD = $r^{1/2}/(1-r) = \sqrt{2}$ =1.41

Note: Q length is non negative. Is mean +- 2 SDs useful?

How long does a customer spend in the system, on average?

If noone in Q, average time is λ_S^{-1} = 1/6 hr or 10 minutes.

But formula page 298 gives $\lambda_S^{-1}/(1-r)$ = 1/3 or 20 minutes.

Section 7.7
Balance Equations for general Markov Q process:
(arrival and service rates can depend on state)
Rate of an arrival coming to state j =
Rate of a service completion from state j+1
We had for the "constant-rates" Markov queue
$\pi_i \lambda_A = \pi_{i+1} \lambda_S$

This can be extended to

$\pi_i a_j = \pi_{i+1} s_{j+1}$

When would the arrival rate depend on the number in the queue? When would the service rate depend on the number in the queue?

See condition on a_j and s_{j+1} for steady state to exist. (7.7-1 p 307)

$1+a_0/s_1+ a_0a_1/s_1s_2 + a_0a_1a_2/s_1s_2s_3 + \ldots$ must be finite. (*)

Sometimes is a summable series like GP

Can then get explicit formulas for the steady state probs for the queue length. See p 308.

Even when explicit sums of the series (*) are not possible, a finite number of terms will do in the expression for π_0 (if it converges), and then all the π_i can be found.

An example of this computation is given in applying these general queuing formulas to the M/M/k queue. See p 309-310.

7.8 Finite Capacity Queues

The infinite series for needed for π_0 becomes a finite series. In this case, any empirical values for a_i or s_i can be used.

Example: capacity 3 queue, M/M/1

Arrival rate = 1 when queue size is 0, 1,
= 1/2 when queue size is 2
= 0 when queue size is 3

Service rate = 2 when queue size is 1,2,3

Then $\pi_0 = (1+1/2+1/4 +1/16)^{-1} = .55$ and
$\pi_1 = .55 * (1/2) = .28$
$\pi_2 = .55 * (1/4) = .14$
$\pi_3 = .55* (1/16) = .03$
and these add to 1.

The following is a copy of the e-mail I sent you over the weekend. It is an example of a project that arises from a hobby of mine – cooking. I hope that you can think of some original project that is based on something that interests you. More detailed guidelines for the project have been posted previously (and were included in the e-mail referred to above.)

What do probability models and simulation have to do with cooking?

Well, suppose you are frying potatoes - you cut them up into little slices and keep turning them over over a high heat. Now from the point of view of a particular piece of potato, the time until it is turned is random, and the temperature it happens to sit on at that particular spot on the pan is also variable. So the time between flips might be exponential with mean m and the temperature might be normal with mean mu and SD sigma. The time it takes to cook a piece of potato might be a function of the product of the temperature and the time, and we might consider this product has to be c for a perfectly cooked piece. The challenge is to simulate the experience of many pieces so that we can figure out the time needed to so that 50% are cooked perfectly or more. Presumably this would also be the best we can do without burning some pieces or having some underdone.

Now this is fairly easy to simulate once reasonable values are chosen for m, mu, sigma, and c. From an informal knowledge of cooking potatoes, I would say that it takes about 10 flips and 5 minutes to cook. So each inter-flip time would be about 0.5 minutes (close enough to 5/11) and we set m =.5. The heat might vary by 50 degrees typically and average 400 F degrees, so I would set mu=400 and sigma = 50. And it looks like c should be about 5*400 =2000 (in minutes-degrees).

To simulate, concentrate on the experience of one piece. Generate about 25 exponentials with mean 0.5 so that the sum is quite sure to be over 5. We will also need 25 normals with mean 400 and SD 50. Then we multiply the exponentials by the normals and compute the partial sum of these 25 products. This piece will be perfectly done when this partial sum hits 2000.

To get the time required for 50% have this sum greater or equal to 2000, we need to simulate the experience of 100 pieces or so look at the distribution of the time until 50 have reached or exceeded 2000.

This last step could be dome by scanning the outcome of the simulation, or by programming it as well as the simulation itself.

Turns out that it takes about 5.6 minutes for half of the potato pieces to be cooked. By varying the % from 50%, one can show that as the time goes from 5.2 to 6.0 minutes, the % cooked goes from 20% to 80%. So you have to watch those potatoes carefully near the end to get them just right!

OK. That is the kind of thing I am looking for. Do you have a hobby? Driving, golfing, reading novels, jogging, ... Or a burning interest in life contingencies, lotteries, or the stock market? There is apparent randomness everywhere. See if you can think of an original application in some area that interests you.
