STAT 280 Dec. 3, 2001

Today: More review. Probability models, relationships between them, and combinations of them. Role of simulation.

Notice: Assignment 5 will be returned at tutorial today (go to the 530 tutorial in AQ 5014 if you do not have a scheduled tutorial today). Farouk Nathoo will also have projects to return for those who missed his Friday 12-1 office hour last week. If you have missed this time too – pick up from me
AQ 10522, Wed 11am-1pm, Friday 11am-1pm.
I will have office hours then too.

Discrete Random Variables:

Discrete Uniform on 1,2,…,n	(n+1)/2	(n+1)(n-1)/12
Bernoulli p	p	p(1-p)
Binomial n,p	np	np(1-p)
Geometric p	1/p	$(1-p)/p^2$
Negative Binomial k, p	k/p	$k(1-p)/p^2$
Poisson m	m	m
Hypergeometric N,n,r	n(r/N)	n(r/N)(1-r/N)c

Where c is finite population correction factor c = (N-n)/(N-1)

Multinomial $p_1, p_2, \ldots, p_r$ $np_1, np_2, \ldots, np_r$

Componentwise variance same as Binomial

Need to know the pmfs of these models.

What about CDFs?

Continuous Random Variables:

Continuous Uniform a,b	$(a+b)/2$	$(b-a)^2/12$
Exponential λ or m	$1/\lambda$	$1/\lambda^2$
Gamma α, β	$\alpha\beta$	$\alpha\beta^2$
Normal μ, σ	μ	σ
Weibull α, β	$\alpha\Gamma(1+1/\beta)$	see p 256
Beta a, b	$a/(a+b)$	$ab/[(a+b+1)(a+b)^2]$

Need to know the pdfs of these RVs, and where available the cdfs as well.

Uses:

Individually:

Discrete uniform: random sampling from list
Bernoulli: 0-1 event
Binomial: n indept 0-1 events, count 1s
Geometric: trials until first success
Neg. Bin.: trials until kth success
Poisson: Count of number of events
Hypergeo: n draws without replacement, number of one kind.
Multinomial: vector of counts of r types from sample of size n.

Continuous Uniform: simulating X with $F_X()$
Exponential: duration under constant hazard
Gamma: duration for k events under constant hazard
Normal: totals, averages, or generalizations of these.
Weibull: durations under non-constant hazard
Beta: random quantities on fixed interval (e.g. probabilities on (0,1).

Relationships:

Bernoulli trials: Bernoulli, Binomial, Geometric, Negative Binomial

Sums and averages: Normal, Negative Binomial, Gamma

Durations (Discrete): Geometric, Neg. Binom.
(Continuous): Exponential, Gamma, Weibull
Counts: Binomial, Poisson, Compound Poisson

Stochastic Processes: (Discrete Time) Bernoulli Trials, Random Walk, Bernoulli Counting Process, Bernoulli Queuing Process, Markov Chain.
(Continuous Time): Poisson Process, Compound Poisson Process, M/M/k Queuing Process.

More models that combine simple models:

Project Examples: such as

Potato-cooking
Elevator operation
Traffic Accidents
Insurance Company Liquidity
Gambling
Electronic Games
Stock Market
Etc.

Queuing process with non-exponential interarrival or service times
Gamblers Ruin
Random Walk Models

Comments on Ch 7 and Assignment 5

7.8-4 (14 telephones)

Queue is constrained to be ≤ 14. A 14 server queue does not in general have a constrained queue length, but in this exercise it is constrained.
This changes the probabilities for the queue-length distribution – for example, length 13 is much more likely in the constrained queue than in the unconstained queue.

Example 7.8-2 shows how to deal with this situation. It does use the formula on p. 308, which is general for M/M/k queues, but the special case resulting in the formulas on p 310 do not apply to a constrained queue.

Ex 7.9-4 just required proper use of the program I provided. The parameter of the exponential in that program required a mean to be specified not a rate parameter (it is easy to get these confused – be careful!). So a rate of 20 per hour would be specified as a mean of .05 hours or 3 minutes. This specification determines the units of the answer. Units must be included in the answer.
(Words too!).

Median score on Project: (70 received) 18/20
Median score on Asst 5: (85 received) 5/10