STAT 280 Term Test Nov. 7, 2001

Instructions: Attempt all questions. This test is open book and notes. Calculators are allowed. You have 60 minutes to complete the test. The marks allotted add to 60.

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1. (25 marks)

For each of the scenarios below, suggest a probability model for the random variable described. Identify the distribution by name or formula, and give specific values for the parameters that suit the scenario. Justify your choice of model and parameter values. (In your answer, you may refer to the text for parameter definitions, if you find this saves time).

a) You arrive at the doctor's office and there 4 patients ahead of you. Let X be the number of minutes you will have to wait before you are moved to an examination room.
b) Sheets of plywood are produced at a mill at the exact rate of one per minute. Each sheets is inspected for defects and on average, one sheet in one hundred is found to contain defects. Let X be the number of defective sheets detected in one hour.
c) Students of a Statistics class of 110 students are surveyed to determine the proportion that have the use of a laptop computer during the course. The survey is based on a random sample of 25 students and X is the proportion of these that do have use of a laptop.
d) Bolts of lightning occur during a thunderstorm at what is described as a "steady rate" over a 15 minute period. Let X be the time from a clap of thunder until the next lightning bolt.
e) Five coins are weighted so that P(Head) on tossing the coin is .4, .45, .5, .55, .6 respectively. One of the five coins is selected at random and tossed 10 times. Let X be the number of heads in these ten throws. (no calculations necessary).

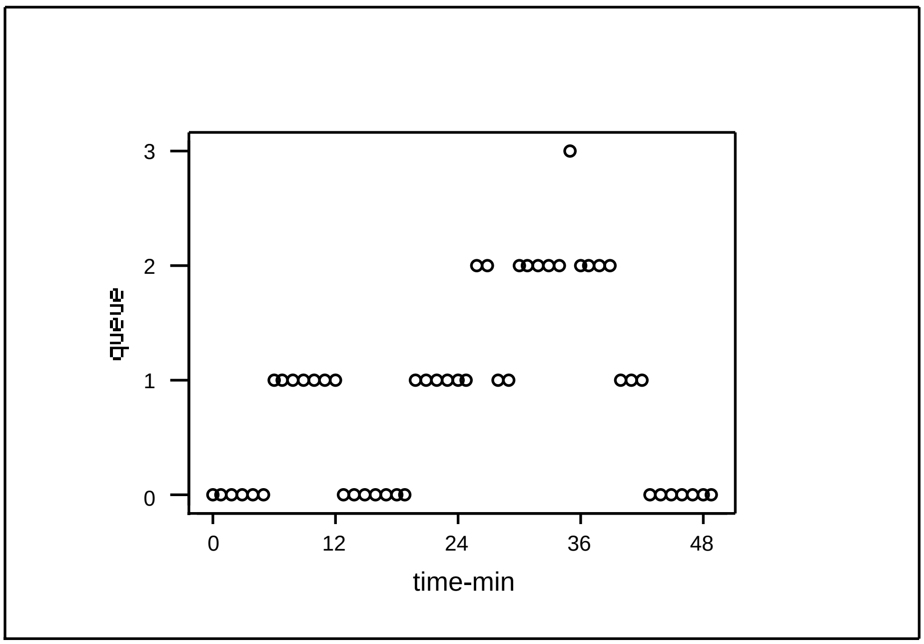
A1. a) Gamma (4, 10) where the average appointment duration has an exponential distribution with mean of 10 minutes. (using alpha, beta as defined on p 251).
b) A Bernoulli counting process with p=.01 and 60 frames of one minute each, so X is the same random variable as X(60) in this Bernoulli counting process.
c) Y is Hypergeometric with number in population, N = 110, and r, number in population with laptop use = 30, and n = sample size = 25 (Reference p 117).. And the proportion X would then be Y/25.
d) The time between lightning bolts is modeled as an exponential with mean 0.5 minutes. The time from any random time to the next lightning bolt is also exponential with mean 0.5 (by the memoryless property of the exponential).
e) The conditional distribution of the number of heads given the coin has P(head) = p is Bin(10;p) (as defined on p 103). And p has a discrete uniform distribution on .4, .45, .5, .55, .6.

2. (8 marks) Customers arrive a barber shop at the constant average rate of 3 per hour. At the end of an eight hour day, what is the chance than fewer than 15 customers had arrived that day?

A2. Use the normal approximation to a Poisson with mean 24. Its SD is $\sqrt{24}$ = 4.9, so 14.5 is 9.5/4.9 = 1.94 SDs below the mean, so the probability is about .03 (Exact cumulative Poisson Probability 0 to 14 is about .0321).

3. (15 marks)

The graph below shows a discrete time queuing process over 50 one-minute time frames.
a) Infer the inter-arrival and service times for the time period shown.
b) Find a point estimate of the inter-arrival and service rates.



A3. a) The arrival times of 5,19,25,29, and 34 give interarrival times of 5,14,6,4, and 5. The corresponding service times are 7,8,8,4, and 3.
b) In 50 minutes, there were 5 arrivals, so the arrival rate is estimated to be 0.1 per minute. The service times averaged 6.0, so the service rate is estimated as 1/6 per minute.

4. (12 marks)

a) Describe the long run behavior of a Markov Chain with the following one-step transition matrix. Assume the initial state X(0)=1, and the state space is {1,2,3,4,5}. No calculations are required - simply examine the one-step transition matrix and comment on the long run behaviour of the chain.

P=

.4	.5	.1	.0	.0
.2	.8	.0	.0	.0
.1	.0	.0	.1	.8
.0	.0	.9	.1	.0
.0	.0	.2	.8	.0

b) Suggest three methods for computing the long run probabilities of the chain.

A4. a) The chain typically spends periods of approximately 10 steps in states {1,2} and then cycles through states 3->5->4->3 with occasional variations in this sequence, for about 10 steps, and then moves back to 1 and repeats the pattern.

b) 1. By simulating the Markov chain and calculating the proportion of time spent in each state over a long realization of the chain

2. By solving the equations xP=x for the 5-unknown probabilties in the vector x (See Thm 4.6-3, p 171)

3. By computing P^k for large k until the columns are constant and filled with the long run probabilities.