

## Student Presentations Today:

Linnea Duke	12.6	Correspondence Analysis	14-Nov
Elizabeth Juarez	10.4	Interpreting Canonical Correlation	14-Nov

If time permits:

Intro to Ch 7: Multivariate Linear Regression

Familiar Multiple Regression model – box p 356 and Result 7.1 p 358

Note assumed independence of rows of the  $n \times p$  data matrix. Also  $Z$  fixed.

$Y = Z\beta + \varepsilon$  and estimate  $\beta$  with  $\hat{\beta} = (Z'Z)^{-1}Z'y$  Can compute SSR, SSE.

Connection with Anova – p 357 – use of dummy variables.

Note: as example p 357,  $Z$  is not rank 4 and  $Z'Z$  is singular. In this case replace  $(Z'Z)^{-1}$  by the generalized inverse  $(Z'Z)^-$  ( $B$  is gen inv of  $A$  if  $BAB=B$ ) p 418 in J-W

More detail at

Eric W. Weisstein. "Moore-Penrose Matrix Inverse." From *MathWorld*--  
A Wolfram Web Resource. <http://mathworld.wolfram.com/Moore-PenroseMatrixInverse.html>

```
y=c(9,6,9,0,2,3,1,2)
```

```
a=c(rep(1,11),rep(0,8),1,1,rep(0,8),1,1,1)
```

```
z=matrix(a,ncol=4,nrow=8)
```

```
> z
```

```
      [,1] [,2] [,3] [,4]
```

```
[1,]  1  1  0  0
```

```
[2,]  1  1  0  0
```

```
[3,]  1  1  0  0
```

```
[4,]  1  0  1  0
```

```
[5,]  1  0  1  0
```

```
[6,]  1  0  0  1
```

```
[7,]  1  0  0  1
```

```
[8,]  1  0  0  1
```

```
> tzz=t(z)%*%z
```

```
> bhat=ginv(tzz)%*%t(z)%*%y
```

```
> bhat
```

```
      [,1]
```

```
[1,]  2.75
```

```
[2,]  5.25
```

```
[3,] -1.75
```

```
[4,] -0.75
```

Note  $2.75+5.25 = 8 = \text{mean in pop 1}$

$2.75-1.75 = 1 = \text{mean in pop 2}$

$2.75-0.75 = 2 = \text{mean in pop 3}$

Connection between regression (lm) and analysis of variance (aov)  
g=cbind(d1,d2,d3)

```
> a=lm(y~g)
> summary(a)
```

Call:  
lm(formula = y ~ g)

Residuals:  
Min 1Q Median 3Q Max  
-4.00 -1.25 0.50 2.00 2.00

Coefficients:  
Estimate Std. Error t value  
(Intercept) 10.0000 2.1180 4.721  
g -3.0000 0.9718 -3.087  
Pr(>|t|)  
(Intercept) 0.00325 \*\*  
g 0.02147 \*

---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.38 on 6 degrees of freedom  
Multiple R-Squared: 0.6136, Adjusted R-squared: 0.5492  
F-statistic: 9.529 on 1 and 6 DF, p-value: 0.02147

```
> b=aov(y~g)
> summary(b)
```

Df Sum Sq Mean Sq F value  
g 1 54.000 54.000 9.5294  
Residuals 6 34.000 5.667  
Pr(>F)  
g 0.02147 \*

Residuals  
---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Note  $9.529 = (-3.087)^2$

See last line of p 357: regression includes aov.

Sampling properties of  $\hat{\beta}$  (pp 363-365)

Result 7.2 (p 363) mean and cov of  $\hat{\beta}$ .

Result 7.3 (Gauss Theorem) about minimum variance linear unbiased estimators.

(Do we need unbiasedness?) "Justifies" plugging in  $\hat{\beta}$  for  $\beta$  in linear predictor.

More next time ....