Answers to Assignment 4 (62, 92, 112, 116a)

62.

a. $E(X) = \alpha\beta = n\dfrac{1}{\lambda} = \dfrac{n}{\lambda}$; for $\lambda = .5$, $n = 10$, $E(X) = 20$

b. $P(X \le 30) = F\left(\dfrac{30}{2};10\right) = F(15;10) = .930$

c. $P(X \le t) = P(\text{at least } n \text{ events in time } t) = P(Y \ge n)$ when $Y \sim$ Poisson with parameter λt.

Thus $P(X \le t) = 1 - P(Y < n) = 1 - P(Y \le n-1) = 1 - \displaystyle\sum_{k=0}^{n-1} \frac{e^{-\lambda t}(\lambda t)^k}{k!}$.

92.

a. $P(10 \le X \le 20) = \dfrac{10}{25} = .4$

b. $P(X \ge 10) = P(10 \le X \le 25) = \dfrac{15}{25} = .6$

c. For $0 \le X \le 25$, $F(x) = \displaystyle\int_0^x \frac{1}{25}\,dy = \frac{x}{25}$. $F(x) = 0$ for $x < 0$ and $= 1$ for $x > 25$.

d. $E(X) = \dfrac{(A+B)}{2} = \dfrac{(0+25)}{2} = 12.5$; $Var(X) = \dfrac{(B-A)^2}{12} = \dfrac{625}{12} = 52.083$

112.

a. $F_Y(y) = P(Y \le y) = P(60X \le y) = P\left(X \le \dfrac{y}{60}\right) = F\left(\dfrac{y}{60\beta};\alpha\right)$. Thus $f_Y(y)$

$= f\left(\dfrac{y}{60\beta};\alpha\right)\cdot\dfrac{1}{60\beta} = \dfrac{y^{\alpha-1}e^{\frac{-y}{60\beta}}}{(60\beta)^{\alpha}\Gamma(\alpha)}$, which shows that Y has a gamma distribution with parameters α and 60β.

b. With c replacing 60 in **a**, the same argument shows that cX has a gamma distribution with parameters α and $c\beta$.

116.

a. $F(x) = \lambda e^{-\lambda x}$ and $F(x) = 1 - e^{-\lambda x}$, so $r(x) = \dfrac{\lambda e^{-\lambda x}}{e^{-\lambda x}} = \lambda$, a constant (independent of X); this is consistent with the memoryless property of the exponential distribution.