SOLUTIONS TO ASSIGNMENT #7

4.

a. $58.3 \pm \frac{1.96(3)}{\sqrt{25}} = 58.3 \pm 1.18 = (57.1, 59.5)$

b. $58.3 \pm \frac{1.96(3)}{\sqrt{100}} = 58.3 \pm .59 = (57.7, 58.9)$

c. $58.3 \pm \frac{2.58(3)}{\sqrt{100}} = 58.3 \pm .77 = (57.5, 59.1)$

d. 82% confidence $\Rightarrow 1-\alpha = .82 \Rightarrow \alpha = .18 \Rightarrow \alpha/2 = .09$, so $z_{\alpha/2} = z_{.09} = 1.34$ and the interval is $58.3 \pm \frac{1.34(3)}{\sqrt{100}} = (57.9, 58.7)$.

e. $n = \left[\frac{2(2.58)3}{1}\right]^2 = 239.62$ so n = 240.

10.

a. When n = 15, $2\lambda\sum X_i$ has a chi-squared distribution with 30 d.f. From the 30 d.f. row of Table A.6, the critical values that capture lower and upper tail areas of .025 (and thus a central area of .95) are 16.791 and 46.979. An argument parallel to that given in Example 7.5 gives $\left(\frac{2\sum x_i}{46.979}, \frac{2\sum x_i}{16.791}\right)$ as a 95% C. I. for $\mu = \frac{1}{\lambda}$. Since $\sum x_i = 63.2$ the interval is (2.69, 7.53).

b. A 99% confidence level requires using critical values that capture area .005 in each tail of the chi-squared curve with 30 d.f.; these are 13.787 and 53.672, which replace 16.791 and 46.979 in **a**.

c. $V(X) = \frac{1}{\lambda^2}$ when X has an exponential distribution, so the standard deviation is $\frac{1}{\lambda}$, the same as the mean. Thus the interval of **a** is also a 95% C.I. for the standard deviation of the lifetime distribution.

20. Because the sample size is so large, the simpler formula (7.11) for the confidence interval for p is sufficient.

$$.15 \pm 2.58\sqrt{\frac{(.15)(.85)}{4722}} = .15 \pm .013 = (.137, .163)$$

36. n = 26, $\bar{x} = 370.69$, s = 24.36; $t_{.05,25} = 1.708$

a. A 95% upper confidence bound:

$$370.69 + (1.708)\left(\frac{24.36}{\sqrt{26}}\right) = 370.69 + 8.16 = 378.85$$

b. A 95% upper prediction bound:

$$370.69 + 1.708(24.36)\sqrt{1 + \frac{1}{26}} = 370.69 + 42.45 = 413.14$$

46.

a. Using a normal probability plot, we ascertain that it is plausible that this sample was taken from a normal population distribution.

b. With s = 1.579 , n = 15, and $\chi^2_{.05,14} = 23.685$ the 95% upper confidence bound for σ is $\sqrt{\dfrac{14(1.579)^2}{23.685}} = 1.214$

a. $P(\min(X_i) \le \tilde{\mu} \le \max(X_i)) = 1 - P(\tilde{\mu}, < \min(X_i) \, or \, \max(X_i) < \tilde{\mu})$
$= 1 - P(\tilde{\mu}, < \min(X_i)) - P(\max(X_i) < \tilde{\mu})$
$= 1 - P(\tilde{\mu} < X_1, \ldots, \tilde{\mu} < X_n) - P(X_1 < \tilde{\mu}, \ldots, X_n < \tilde{\mu})$
$= 1 - (.5)^n - (.5)^n = 1 - 2(.5)^{n-1}$, from which the confidence interval follows.

b. Since $\min(x_i) = 1.44$ and $\max(x_i) = 3.54$, the C.I. is (1.44, 3.54).

c. $P(X_{(2)} \le \tilde{\mu} \le X_{(n-1)}) = 1 - P(\tilde{\mu}, < X_{(2)}) - P(X_{(n-1)} < \tilde{\mu})$
= 1 – P(at most one X_i is below $\tilde{\mu}$) – P(at most one X_i exceeds $\tilde{\mu}$)
$1 - (.5)^n - \binom{n}{1}(.5)^1(.5)^{n-1} - (.5)^n - \binom{n}{1}(.5)^{n-1}(.5)$
$= 1 - 2(n+1)(.5)^n = 1 - (n+1)(.5)^{n-1}$
Thus the confidence coefficient is $1 - (n+1)(.5)^{n-1}$, or in another way, a $100(1 - (n+1)(.5)^{n-1})\%$ confidence interval.

58.

a. $P(\min(X_i) \le \tilde{\mu} \le \max(X_i)) = 1 - P(\tilde{\mu}, < \min(X_i) \, or \, \max(X_i) < \tilde{\mu})$
$= 1 - P(\tilde{\mu}, < \min(X_i)) - P(\max(X_i) < \tilde{\mu})$
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b. Since $\min(x_i) = 1.44$ and $\max(x_i) = 3.54$, the C.I. is (1.44, 3.54).

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Thus the confidence coefficient is $1 - (n+1)(.5)^{n-1}$, or in another way, a $100(1 - (n+1)(.5)^{n-1})\%$ confidence interval.