#10

Population Strength Distribution is $N(\mu,\sigma=60)$ - units are KN/m^2

a) H_0: $\mu = 1300$

H_a: $\mu > 1300$

(This is tricky – read question carefully – we are trying to prove $\mu >$ 1300, and we are doing this by `testing credibility of H_0)

Rejection region is specified as $\overline{X} \geq 1331.26$

P(type I error)= $P(\overline{X} \geq 1331.26)$ assuming H_0 true.

= $P(z \geq ((1331.26-1300)/13.416))$

= $P(z \geq 2.33) = 1-0.9901 = .01$

c) With the assumption $\mu = 1350$, $P(\overline{X} \leq 1331.26) = P(z \leq -1.40) = .081$

d) We need to change the rejection region $\overline{X} \geq c$ so that $P(\overline{X} \geq c) = .05$ where $z = (c-1300)/13.416 = 1.645$

(since N(0,1) table has $P(z \geq 1.645) = .05$). Solving for c=1322.07.

By choosing this larger type I error rate (.05 compared to .01 in part b)) we can reduce the type II error rate (which was .081 in part c)) to $P(\overline{X} \leq 1322.07)$ when $\mu = 1350$, which is $P(z \leq (1331.26-1350)/13.416)$ $= P(z \leq -2.08) = .0188$

e) $z \geq 2.33$ (see calculation in part b)).

26. Reject H_o if $z \geq 1.645$; $\frac{s}{\sqrt{n}} = .7155$, so $z = \frac{52.7-50}{.7155} = 3.77$. Since 3.77 is ≥ 1.645, reject H_o at level .05 and conclude that true average penetration exceeds 50 mils.

27. We wish to test H_o: $\mu = 75$ vs. H_a: $\mu < 75$; Using $\alpha = .01$, H_o is rejected if $t \leq -t_{.01,41} \approx -2.423$ (from the df 40 row of the t-table). Since $t = \frac{73.1-75}{5.9/\sqrt{42}} = -2.09$, which is not ≤ -2.423, H_o is not rejected. The alloy is not suitable.

40.

a. P = true proportion of current customers who qualify. H_o: p = .05 vs H_a: p ≠ .05,

$z = \frac{\hat{p} - .05}{\sqrt{.05(.95)/n}}$, reject H_o if z ≥ 2.58 or z ≤ -2.58. $\hat{p} = .08$, so

$z = \frac{.03}{.00975} = 3.07 \geq 2.58$, so H_o is rejected. The company's premise is not correct.

b. $\beta(.10) = \Phi\left[\frac{.05 - .10 + 2.58\sqrt{.05(.95)/500}}{\sqrt{.10(.90)/500}}\right] = \Phi(-1.85) = .0332$

52.

a. For testing H_o: p = .2 vs H_a: p > .2, an upper-tailed test is appropriate. The computed Z is z = .97, so p-value = $1 - \Phi(.97) = .166$. Because the p-value is rather large, H_o would not be rejected at any reasonable α (it can't be rejected for any α < .166), so no modification appears necessary.

b. With p = .5, $1 - \beta(.5) = 1 - \Phi[(-.3 + 2.33(.0516))/.0645] = 1 - \Phi(-2.79) = .9974$