

**Today: Follow-up from last day's lecture**

**Risk: Insurance (connection with randomness)**

**Risk: Investment (variability and rate of return)**

**Follow-up from last day:** 1. experiments to prove causal links & randomization  
2. detecting real effects from illusions

Why are these strategies necessary?

1. hidden causes in observational studies ("lurking variables")
  - e.g. smoking and serious disease
  - e.g. sex and graduate admissions at Berkeley
2. Intuition not good in assessing chance effects
  - e.g. sports leagues
  - e.g. stock price charts

**Next topic ....?**

Where are we? Keep a copy of the course outline in your reader. The first three topics (1.,2.,3.) are done. Today we start on topic 4: "**Risk**".

We will investigate certain aspects of risk in insurance and investment. First we need to understand what risk is...

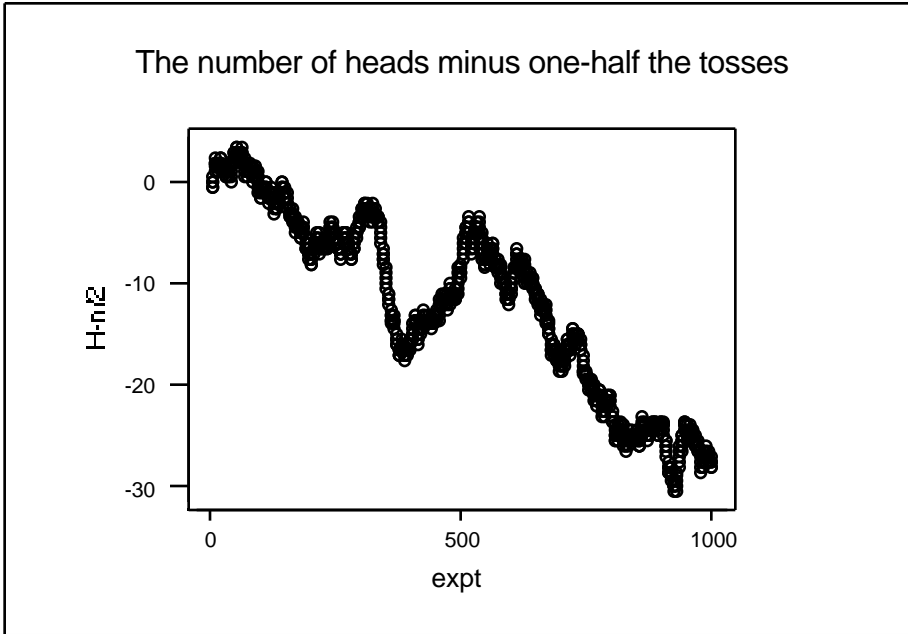
One dictionary definition of **risk** is "The probability of loss". But what is **probability**? Think of a sequence of identical and independent experiments in which a certain event can happen in each experiment, but may not happen - like getting heads in a sequence of coin tosses. Other examples....?

Now consider the proportion (=relative frequency) of those experiments in which a certain event occurs - like the proportion of tosses resulting in heads. This proportion will stabilize as we increase the number of experiments, and the number that it stabilizes at is called the probability of the event.

Probability of an event is therefore the **long run relative frequency** of the event in a sequence of independent and identical experiments.

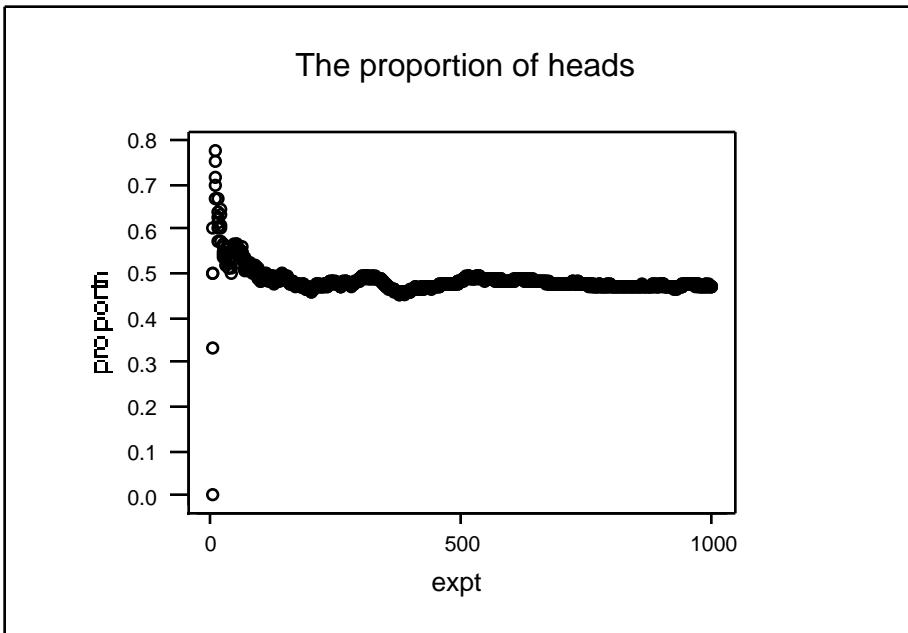
Consider 1000 tosses of a fair coin. How does the number of heads compare with one-half the number of tosses?

Simulation of H (1) and T(0): 1,1,1,1,0,1,0,0,1,1,0,.....for H,H,H,H,T,H,T,T,H,H,T,...



Why does the graph appear to drift downward when the coin is fair?

Lets look at what happens to the proportion of heads:



Not doubt in this case that the proportion is stabilizing at 0.5 as the number of tosses grows.

Is there something we can say about what will happen in a single throw?  
 Can't predict outcome but can say "Probability of a head" = 0.5

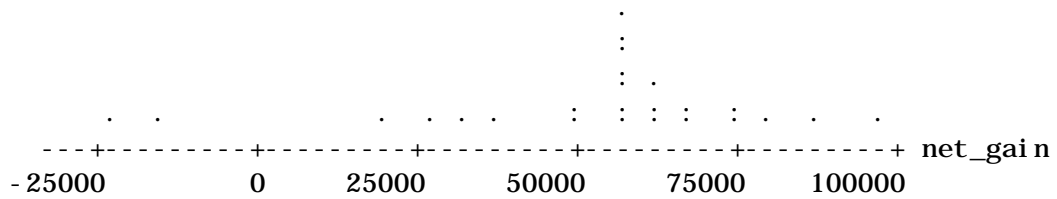
Now reconsider: **risk** is "The probability of loss"

Suppose we buy a lottery ticket. Is this a risky investment?

Now consider **Insurance**:

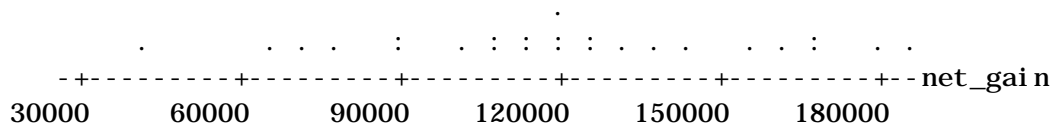
When we drive our car to SFU, is this risky? The cost of an accident can be large. Often we would be willing to pay a little each time in order to avoid a big loss if an accident occurs. Suppose we pay \$5 every day for auto insurance. The company receives  $5 \times 365 = \$1825$  in one year. If I have no accident the company keeps the \$1825. If I have an accident, suppose the average cost is \$6,000. Also, suppose the company has determined that my probability of having an accident this year is  $1/5=0.2$ . will the company make money?

With one customer, the company could not be sure. But with 100 customers, here is what would happen according to several simulations of a years experience with 100 customers:



Not too risky?  $P(\text{loss}) = 2/25 = .08$  (8%)

What if 200 customers?



$P(\text{loss}) = 0/25 = 0$  ?

The company is making a gross profit of about \$120,000 or \$600 per customer.

Pay for overhead?

Insurance is a way to "spread the risk". The company providing the mechanism for doing this charges for the service. People are willing to pay for this.

Next: **Investment:**

Consider buying a stock (partial ownership of a company)?

Say 100 shares of GM (among millions that others own).  $P(\text{loss})=?$

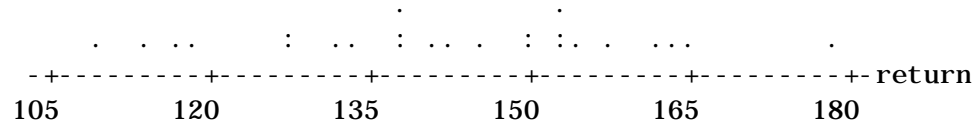
Obviously some stocks are more "risky" than others - larger probability of a loss.

Is it possible to use risky stocks to make money reliably?

Suppose I hold 1 shares in each of 100 companies, and suppose all the companies have shares priced at \$1.00, so my total investment is \$100. Further suppose that each company has the following prospects over the following year. The amount under "prospect" below is the possible share market value at the end of the year.

Prospect	Probability
\$0.00	0.25
\$0.50	0.25
\$1.00	0.25
\$4.00	0.25

These 100 companies are all very risky in the sense that  $P(\text{loss}) = .5$  and also considering  $P(\text{gain}) = 0.25$ . But suppose the companies have nothing to do with each other so the success or failure of one is unrelated to the success or failure of another. Let us investigate what would happen in one year with this portfolio of 1 share in each of 100 companies. I simulated this situation 25 times with the result:



Our \$100 investment in the 100 risky companies has produced a profit in each of the 25 times it was run. Typical profit was about  $140 - 100 = 40$  or about 40% - so risky companies may not be so bad after all.  $P(\text{portfolio loss}) = \text{approx } 0$

Caution: a key assumption here is that the 100 company results are unrelated. This is not usually true but the argument still holds as long as the portfolio is well diversified.

Note: Investment advisors often use the SD of past monthly returns to gauge the "riskiness" of a stock or of a portfolio. Does SD measure "risk"?

Appendix: program that simulates the "risky" stock portfolio:

```
Gmacro
risky.mac
set c1      # set up discrete probability distribution
0,.5,1,4
end
set c2
4(.25)
end
do k1=1:25  # do for 25 yearly experiences
rand 100 c3; #generate the random annual outcomes for 100 companies
discrete c1 c2.
sum c3 k2   # add up the result for the portfolio for one year
let c4(k1)=k2 # store the result in a column
enddo      # end this years experience and do another one
dotplot c4  # dotplot of the 25 experiences of the portfolio results.
endmacro
```

The program is run by typing

`%risky`

after the

MTB>

Prompt in the session window.