STAT 100 Chance and Data Analysis Sept. 23, 2002(Revised)

Today: **Risk in Investment Portfolio, SD of an Average**

Consider buying a stock (partial ownership of a company)?

Say 100 shares of GM (among millions that others own).

Risk = P(loss)=?

Obviously some stocks are more "risky" than others - larger probability of a loss.

Is it possible to use risky stocks to make money reliably?
Suppose I hold 1 share in each of 100 companies, and suppose all the companies have shares priced at $1.00, so my total investment is $100. Further suppose that each company has the following prospects over the following year. The amount under "prospect" below is the possible share market value at the end of the year.

Prospect	Probability
$0.00	0.25
$0.50	0.25
$1.00	0.25
$4.00	0.25

In other words, for each company that I invest $1.00 in, I will get back at the end of the year either $0.00, $0.50, $1.00, or $4.00, and each of these possibilities has the same chance of occurring. If this were the case, then in 75% of the time, on average, I will be wasting my money, because I will be no further ahead. (Putting the $1.00 in the bank might only yield a few cents but that is better than $0.00.) And 50% of the time I will actually lose money from my $1.00 investment. Most people would say this was a very risky investment.

These 100 companies are all very risky in the sense that P(loss) = .5 and also considering P(gain) = 0.25. But suppose the companies have nothing to do with each other so the success or failure of one is unrelated to the success or failure of another. Let us investigate what would happen in one year with this portfolio of 1 share in each of 100 companies. I simulated this situation 25 times with the result:

```
                          .         .
     .  . ..     :  ..  : .. .  : :. .  ...        .
  -+---------+---------+---------+---------+---------+-return
 105       120       135       150       165       180
```

Our $100 investment in the 100 risky companies has produced a profit in each of the 25 times it was run. Typical profit was about 140-100 = 40 or about 40% - so risky companies may not be so bad after all. P(portfolio loss) = 0, approximately.

Note: *Effect of averaging to reduce variability.*

[Caution: a key assumption here is that the 100 company results are unrelated. This is not usually true but the argument still holds as long as the portfolio is well diversified so that there is some independence among the company results.]

Note: Investment advisors often use the SD of past monthly returns to gauge the "riskyness" of a stock or of a portfolio. Does SD measure "risk"?

Recall that the SD depends on the mean (i.e. average) so let's take a look at that first:

The returns are 0, .5, 1, and 4 with equal probability. The mean return is therefore

(0+.5+1+4)/4 = 5.5/4 = 1.375. Lets round to 1.38. This means that, on average, every dollar invested in this company will return $1.38, a 38% gain. But of course the average does happen in one instance - for example the return can be -$1.00 (a loss). The average net return for a $1.00 investment in this company is $0.38.

What about the SD of the net return? (net return = return - $1)
(In class, there was confusion about **return** and **net return** - return is 0,0.5,1,4, net return is -1,-.5, 0, 3.)

The SD of the net return from a single company is the SD of the numbers -$1.00, -$0.50, $0.00, and $3.00.

$(-1-0.38)^2 + (-0.50-0.38)^2 + (0 - 0.38)^2 + (3-0.38)^2$ is the sum of the squared deviations
$=1.38^2 + 0.88^2 + 0.38^2 + 2.62^2 = 9.69$
So the average squared deviation is 9.69/4 = 2.42 approx
And the SD = the square root of 2.42 = 1.56

The units of this are dollars. The SD measures a typical deviation from the mean and in this case it is $1.56 - for an average of $0.38 to have a typical deviation of $1.56 is pretty alarming! Yes, it does seem that these companies are a risky investment since the probability of loss is quite high (50%). Another way of judging this would be to say that the SD of the return is large compared to the mean, so that losses are quite likely.

But note that the investment in 100 of these companies (assuming independence) has very little risk. Probability of loss is almost 0. How can we resolve this confusion?

Let's compute the mean and SD of the return on 100 different stocks just like the one risky one.

Intuitively, the average must still be 1.38 per company, or $138 for the portfolio. What about the SD? It turns out that SD of the average return is the SD of the individual return divided by the square root of number of things averaged, n (of 100 in this case).

In symbols SD(of mean) = SD(of individual)/ $\sqrt{n}$
In other words the variability of the average return should be about 1.56/10 = .156

But look at the simulated result again:

```
                                .         .
     .  . ..     :  ..  : .. .  : :. .  ...        .
 -+---------+---------+---------+---------+---------+-return
105       120       135       150       165       180
```

These are returns, and the average return would just be these divided by 100. The typical deviation of the return looks to be about 15, or in terms of the average .15.
The calculation above shows that it will tend to be .156.

This formula

SD(of mean) = SD(of individual)/ $\sqrt{n}$

is useful because we did not need to do the simulation to get it.

Why does the investor want to know the SD of the portfolio return? It should be obvious …

What have we learned here?

1. Averages are more stable than individual values
2. The SD of an average is less than the SD of the individual values by a factor $\sqrt{n}$
3. In the investment context, 1. and 2. imply that it may be possible to have a secure investment based on a portfolio of risky investments.
4. SD has some relationship to risk, although it is not the same thing (i.e. SD ≠ P(loss))

What would you rather have?
a) a portfolio that returned an average of 10% in one year with a SD of 5%.
b) a portfolio that returned an average of 40% in one year with a SD of 15%.

Based on SD as a measure of risk, we might say option a). But many would be willing to take option b). Note the probability of loss (=risk) is negligible even in a).

While SD has some relation to risk, it is not risk in the sense of Probability of Loss.

How is a simulation of a risky company like the one shown accomplished. Can it be done by coin tossing? Yes.

We need to find a way to get an ordinary coin to tell us which of four options to choose:

0,.5,1, or 4.

Let's do the following: Consider two tosses of a fair coin: Possible outcomes are:

H,H, or H,T or T,H or TT. If the coin is fair, are these outcomes equally likely? If the four outcomes all have the same chance of occurring, what is the chance that a particular one occurs? (A: 25%).

So we can use the four coin outcomes to simulate one company's annual return:

H,H means return of $0
H,T means return of $0.50
T,H means return of $1.00
T,T means return of $4.00

This is what the computer program used to generate the dotplot above - of course, the computer did not really toss a coin, but rather did something that has similar outcomes with similar frequencies. Note that our graph showing the 25 portfolio returns would have required 200 coin-tosses for each point in the graph, or in other words 25 X 200 = 5000 coin tosses.

Appendix: program that simulates the “risky” stock portfolio (This is only for those who choose to try MINITAB - it is optional in this course).

```
Gmacro
risky.mac
set c1        # set up discrete probability distribution
0,.5,1,4
end
set c2
4(.25)
end
do k1=1:25     # do for 25 yearly experiences
rand 100 c3;  #generate the random annual outcomes for 100 companies
discrete c1 c2.
sum c3 k2      # add up the result for the portfolio for one year
let c4(k1)=k2  # store the result in a column
enddo          # end this years experience and do another one
dotplot c4       #  dotplot of the 25 experiences of the portfolio results.
```

endmacro

The program is run by typing

%risky

after the

MTB>

Prompt in the session window.