STAT 100 Chance and Data Analysis Oct 25, 2002(revised)

Today: Tidbit on Investment Portfolios, Risk, and Variability
Lotteries
(Feedback from Wednesday's survey will be detailed on Monday)

Next: Lotteries:

Last 2 digits of your student number can be your lucky number.

Use Cards with A=1, 2-9, 10=0 and select winner of "twoney". If more than one winner, need to pay out more than one twoney.

What should I charge to run this lottery? How much for tickets?

Chance of winning is 1/100. Cost for break-even, **on average**, is 2¢ per ticket.

How much might I have to pay out? Suppose 90 students attending. Each has 1/100 Chance of winning $2. Distribution is same structure as in the childrens' picture recall – The Binomial Distribution. MINITAB provides the following:

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MTB > pdf;
SUBC> binomial 90 .01.
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Probability Density Function

Binomial with n = 90 and p = 0.0100000

x	P(X = x)
0	0.4047
1	0.3679
2	0.1654
3	0.0490
4	0.0108
5	0.0019
6	0.0003
7	0.0000

What happens if nobody wins? Walk away!

There is a 40% chance I get away without ANY payout. BUT, I might have to pay out as much as 4 twoneys ($8). Lets simplify the above a bit:

I pay out $0 with probability	40%
$2	37%
$4	17%

$6	5%
$8	1%
$10 or more	about .2 % (negligible)

How much would I pay out on average?
.4 x0 + .37 x 2 + .17 x 4 + .05 x 6 + .01 x 8 = .74+.68+.3+.08 = $1.80

If I were selling tickets for a break-even lottery, I would need to charge 2¢ per ticket (since 90 x 2¢ = $180). While I cannot tell what would happen in a particular lottery, I can predict that if I were to repeat this lottery many times, my average net profit would be $0.

Does this mean I can be assured of breaking even in the long run? Do I need bankruptcy insurance?

What if I decide to guarantee that at least one prize is given out? I keep drawing two-digit numbers until I get one or more winners. What are the chances then? To do this we just have to allocate the 40% to the rest of the outcomes. The way to do this is to jack up the chances for 1,2,3, …winners ($2, $4, $6, …) by dividing the original chances by .6 (so they still add to 100%).

The new distribution looks like this:

$2	61%
$4	28%
$6	8%
$8	2%
$10 or more	about .3 % (negligible)

Now what is my average cost (per lottery)?

.61 x $2 + .28 x $4 + .08 x $6 + .02 x $8 + … = $1.22 + $1.12 + $0.48 +$0.16 + a bit = $2.98 + a bit or = approx $3.00

So if 90 students bought tickets, we would need an average of 3.00/90 = 3.33¢ per student to break even, on average.

(What if there were 80 students? The average cost in this case is (by going through the entire process again) $2.89 so the break even ticket price would be 2.89/80 = 3.6¢
The fewer students there are, the greater the cost per student of guaranteeing a payout.)

Suppose this were a public lottery, to be repeated many times in a year. If we charge 5¢ for a ticket, and pay out $2 for every winner, do we generate any money for community services?
Note: In class there were about 90 students and the winning number "48" was held by one student, so the payout was a modest $2.

Now consider a lottery like 6/49. Six numbers in the range 1 to 49 are chosen, without replacement, and a perfect match provides a prize of $7,000,000. Let us ignore smaller prizes for now. Since there are 14 million possible choices of six numbers in this range, each ticket has a roughly 1/14,000,000 chance of winning. Suppose tickets cost $1 each. We have to decide what to do if many people choose the same number, and that number wins. Let us suppose that the winners must split the prize in this case. If we sold 14,000,000 tickets, we would have revenues of $14,000,000 and we would pay out $7,000,000. We would definitely be able to pay the ticket sellers and support community services!

Consider the following questions:

1. What is the average return to a $1 ticket?
2. What is the chance that no jackpot is won?
3. If a carry-over jackpot of 15 million dollars is added to the jackpot prize, what is the average return to a $1 ticket in this case?

ANS.
1. 50¢
2. It does depend on the number of tickets sold. If 14,000,000 tickets were sold, the chance that no jackpot is one is about 37% (probability is e^{-1}). If 28,000,000 tickets were sold, the probability falls but only to about 14% (e^{-2}).
In other words, there is a pretty good chance that there will have to be a carry-over of the jackpot prize.
3. It depends on the number of tickets sold. In this situation, it is likely that two or three times the usual number of tickets are sold. Say 42,000,000. Now the lottery corporation would still take its 50% before determining the prize pool, so only $21,000,000 would be returned in prizes from these tickets. But this is added to the $15,000,000 from previous unpaid jackpots, so the total payout is $36,000,000. This divided among the 42,000,000 tickets means the average return to a ticket in this richer lottery is 36/42 *100 or 86¢ per dollar ticket. Still not a great investment except for the hope and entertainment value.

Here is another interesting question about lotteries:

When people can choose their own numbers, what numbers do they choose?

They tend to choose numbers 31 or less (birthdays). They tend to choose odd numbers (not even ones). They tend to avoid 10, 20, 30, 40. Numbers like 7, 11 are popular. They tend to choose combinations that are spread throughout the range 1-49. Would you choose the combination 1,2,3,4,5,6 ?

Why not?

Are there better numbers to choose?

Not much you can do to gain advantage, except to choose unpopular numbers, so if you win, you would not have to share your jackpot. But maybe a lot of people are doing this and the unpopular combinations suddenly become popular!

Does the square root law help to reduce variability of winnings from a lottery?

Yes and No.

If you invest in many public lotteries, or one lottery many times, you will probably still get no major prize but there is a small chance of a large prize. SD will not be small enough to provide a useful prediction.

Average return will be about 50¢ per dollar invested (negative 50¢ net return) which will be realized if you buy a huge number of tickets. (e.g. all the tickets!). Small SD in this case.

Big Picture from this lecture:

1. Lotteries use random sampling
2. Average payout in almost all lotteries is less than ticket cost
3. Carry-over jackpots may still not make a lottery a good investment (avg net return < 0)
4. Exact calculations for a real lottery can be complex, but can be simulated.
5. Square root law does not reduce variability enough to be useful in this context