

Three card problem:

This problem starts with a special 3-card deck:

One card is Black on both sides

One card is Red on both sides

One card is Black on one side and Red on the other side.

Each card is hidden in an envelope and the envelopes are shuffled.

Then you choose an envelope and are allowed to see one side of the card in that envelope.

Suppose you see a red side.

You reason that the card is either the Red-Red card or the Red-Black card.

Because of the shuffling, you reason that the two possibilities are equally likely.

You are offered the following gamble: if the card is Red-Black, you win \$1. If it is Black-Black, you lose \$1.

Is this a fair gamble?

Answer: No. The chance that the card is Red-Red is $2/3$. If a Black had shown, the chance would have also been $2/3$. Clearly, whenever the selected card is a same-coloured card, you lose, and there is a $2/3$ chance that this occurs.

The lesson in this little example is that simple options are not necessarily equally likely, even if they are produced by a random selection mechanism. One must think carefully about the selection process.

There are many other famous versions of this problem. The Monte Hall choice is one of them. Monte Hall had a TV program "Lets make a deal" and this led to the following problem. Here is how wiki describes the problem (http://en.wikipedia.org/wiki/Monty_Hall_problem)

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

The answer is that you should always switch! The reason is that there is a $2/3$ chance that your first choice will be a goat. When this is true, and as soon as the other goat door is revealed, switching will in this case give you the car. In other words, switching gives you a $2/3$ chance of winning the car, not switching a $1/3$ chance.