Stat 280 Mid-Term Test March 6, 1998

Instructions: Attempt all questions. You have 50 minutes to earn 50 marks. Each question is worth 10 marks. You may use any texts or notes.

1. a) Describe in point form the Binomial experiment (i.e. the conditions that give rise to the Binomial distribution)

b) Explain why Binomial probabilities involve the use of a combinatorial coefficient?

A1. a) Bernoulli trials (IID, 0-1 outcomes), X=number of 1s in n trials.

b) Each ordering of given number of 0s and 1s has same chance, and combinatorial coefficient is the number of such orderings with the given mix.

2. Explain the model connections illustrated by the following diagram (I mean the logical connections between the conditions that give rise to these distributions, not the algebraic connections in the formulas):

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Geometric---------------------Exponential
          |                        |
          |                        |
          |                        |
Negative Binomial-----------Gamma
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A2. Geo - number of Bernoulli trials til first success (or 1)

Exp - length of time til first "success" (actually termination of survival time), constant hazard

Gamma - length of time til kth "success" (terminations of survival times), constant hazard

N Bin - number of Bernoulli trials til kth success

3. a) Under what conditions would a negative exponential distribution be approximately symmetrical about its mean?

b) How large does a sample size have to be so that the sampling distribution of the sample mean is well-approximated by the normal distribution, in the case in which the population sampled is normal?

A3. a) Never (Only in the degenerate case when its mean is zero)

b) n=1

4. Customers arrive at a store in an apparently random manner, but with a more-or-less constant rate of 2 per minute.

a) Using a reasonable model, how likely is it that the gap between customers is 2 minutes or more?

b) How long would one have to wait, on average, for 10 customers to arrive?

A4. a) 1-(1-exp(-2*2)) = exp(-4) or about .018

b) The random time has a Gamma distribution with shape parameter alpha=10 and scale parameter Beta=1/2. The mean of this distribution is 1/2 * 10 = 5 minutes (which is what you would expect!)

5. For each I, X(i) = 0,1,2 with equal probability and can take no other values. Let $Y = \sum_{i=1}^{n} (X(i) - 1)$. Suppose the X(i) are mutually independent.

a) What are the mean and standard deviation of Y?
b) Draw a rough sketch of the probability law of Y when n=10.

A5. a) Mean and Var of X are 1 and 2/3, and of X-1 are 0 and 2/3. So mean and SD of Y are 0 and sqrt(2n/3).

b) Approx normal with mean 0 and SD = sqrt(20/3) = approx 2.6 (draw curve - to do this note that point of inflection is at +- 1 SD. Note also that distribution is really a prob mass function since values of Y are discrete.)

(KLW 98/03/06)