System Degeneration and Repair
Relating Model Parameters to Observables

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Introduction:

Health monitoring systems are usually based on information collected during interventions, whether they be planned or unplanned. The black box which represents the health status process for individuals between interventions is both complex and almost unmeasurable. In this paper we propose a model for a process of lifetime health status. The model includes some simple control parameters that approximate the decisions of those that control the health care system. One objective is to try to validate the model by simulating population experience that matches data from interventions. Another objective is to predict the effect of changes in the control parameters.

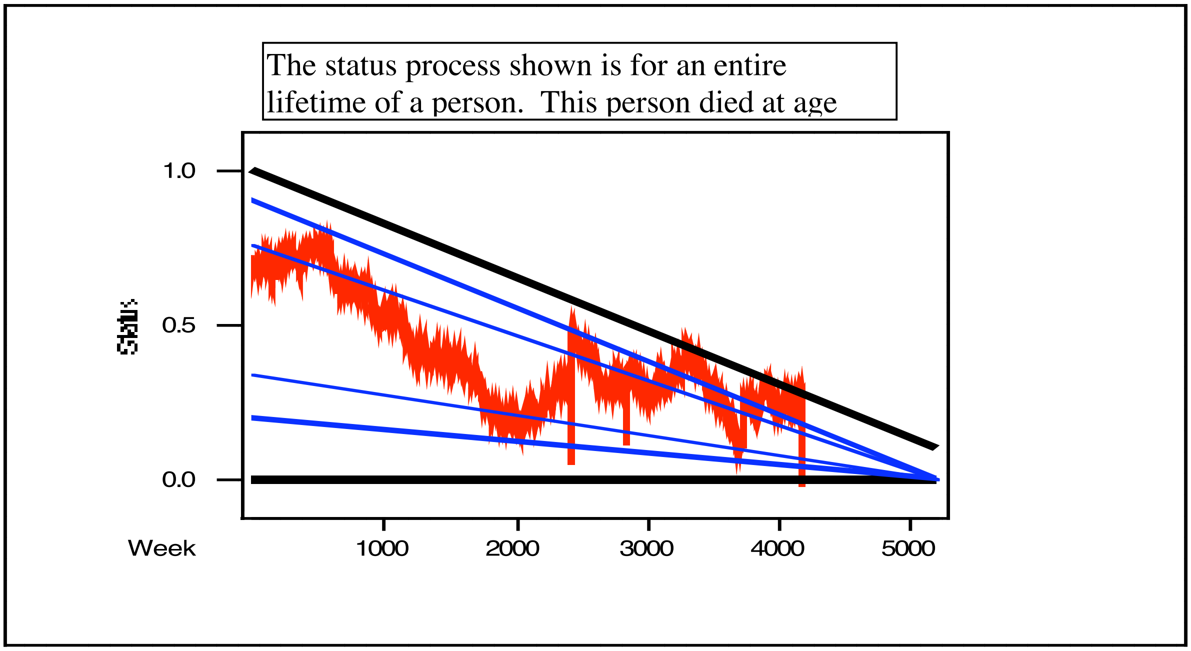
Our approach is to model the stochastic process of a scalar 'health status', with admission to hospital when health status falls to a certain level, and discharge from hospital when the patient achieves a sufficiently high health status. These admit and discharge levels are dependent on age, and also on whether the person has had any previous hospitalization. Because longitudinal data on individuals are usually unavailable over a long period of time, even for the interventions, we assume only that data are available on the interventions experienced during an observation window of a few years. Our stochastic model of the status of individuals in a population, and the implied interventions during an observation window, is then compared to observed data. The model uses numerical representations of the status corresponding to 'perfect' health, death, and a continuum of values in between these extremes.

Our model assumes an alternating phase stochastic process of degeneration and repair, with phase changes determined by the control levels for admission and discharge. The model can be generalized to apply to repairable systems, including machines. The feature of periodic maintenance is important for machines, and arguably may also apply to human populations in view of the "annual checkup". Another feature of the model that complicates the analysis but can be added to the model simulations is the occasional occurrence of relatively large pulses which reduce the health status dramatically, the real-life equivalents of traffic accidents or sudden microbiological anomalies.

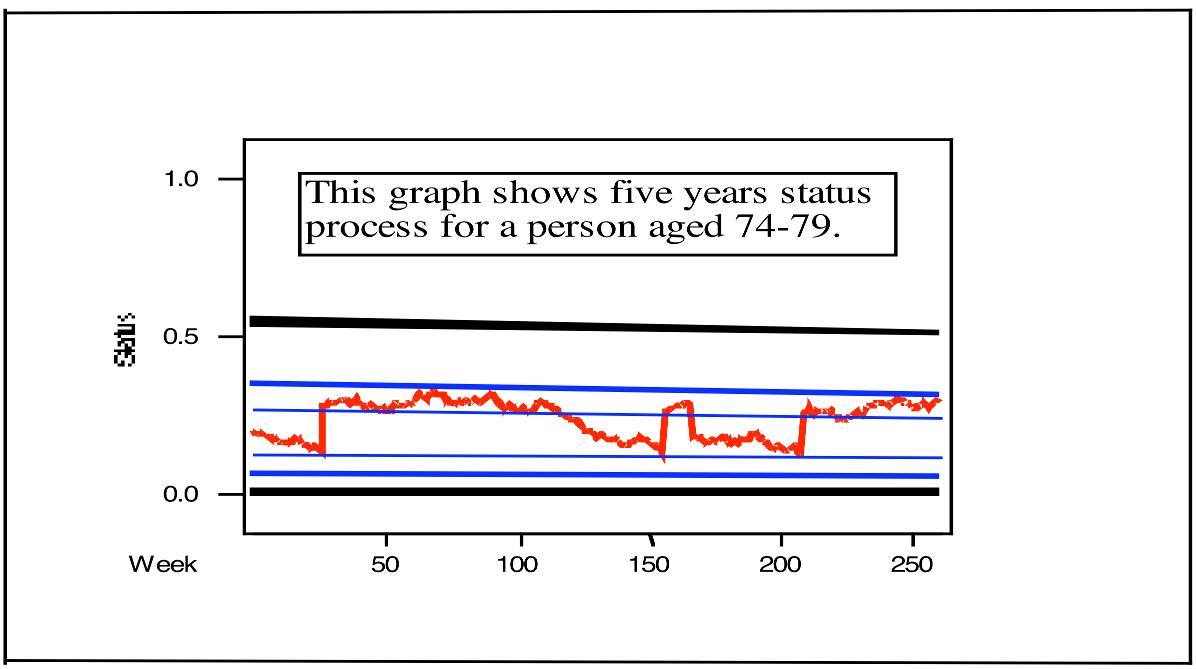
The model:

We first describe the longitudinal stochastic process for health status of an individual over an entire lifetime. Then we use the model to simulate hospitalization experience for each member of a stationary population of individuals over a calendar time window of five years. The population simulated is selected at random from those individuals alive at the beginning of the window. There exist examples of longitudinal hospitalization data over such a time period, and thus it is possible to check that realistic parametrizations of the model are feasible.

The following graph is an example of a lifetime of health status simulated by our model. The details are outlined below:



The health status of individual i at time t is denoted $S_i(t)$. For the purpose of simulation of the model we assume a discrete time parameter representing weeks, so that a five year time window is represented as a time series of 260 points, t=0,1,...,259. At any time t in the window, the age of the ith individual is $A_i(t) = A_i(0)+t$, where the age of the ith individual at the beginning of the observation window is $A_i(0)$.



$A_i(0)$ is simulated either by using a cross-sectional sample of a birth and death process, or by an equivalent procedure. To avoid simulation of individuals not present during the window, we use the following procedure:

i) Specify the population lifetime distribution of the stationary birth-death process. For this we use a Weibull (15, 80) distribution.

ii) Modify the distribution in i) to allow for the effect of cross-sectional sampling: the starting ages in the observation window will be selected from lifetimes according to a bias proportional to the length of the lifetime, L. To achieve this adjustment, one must generate the sample of lifetimes from a density that is a modulation of the original density - one multiplies the density f(x) in i) by the ratio x/μ, where μ is the mean under f().

iii) Simulate the selection of the age at which individuals begin the observation window by using a uniform (0,1) distribution for the proportion of the lifetime spent U, at this moment. Thus $L_i U_i = A_i(0)$.

From time t to time t+1, the health status of an individual will change by an amount Δ_d or and amount Δ_u depending on whether the process is in a "down" phase or an "up" phase. The down phase is a period of degeneration while the up phase is a period of repair. In the context of hospital use, the up phase is a hospital stay. S(t) in a down phase is an asymmetric random walk with step size $\Delta_d \sim N(\mu_d,\sigma_d)$ and in an up phase is another asymmetric random walk with a step size $\Delta_u \sim N(\mu_u,\sigma_u)$.

Although Δ_d and Δ_u have distributions which remain constant over each down or up phase, respectively, it is reasonable to allow them to depend on the age at which the phase begins. In our model we let Δ_u remain constant for all ages, but chose Δ_d in the following way. The individual always starts a down phase with an average decline rate that produces a Status of 0 at age 100 years. Of course, with variability, the actual average life is much less. This rate is reset at the beginning of each down phase, and is determined by the height of the discharge line at the age the down phase begins.

The phase changes are determined by thresholds of S(t) which depend on age. The thresholds as a function of age are lines - the upper one is the discharge line, the status at which the individual is discharged from hospital, and the lower one is the admission line, the status at which the individual is admitted to hospital. An individual will oscillate in an interval roughly determined by these threshold lines until death. Absolute bounds above and below the sample path are the line of "perfect health" and the line of "death". The death line is always at 0.0, but the line of perfect health may be at level 1.0, or else decreasing from 1.0 with time but above the discharge line (parallel to the discharge line).

The simulation has adopted certain strategies for the details of the phase turn-arounds, necessitated by the discretization of the process. In any step in which the status crosses 0, the individual is presumed dead. Any step potentially above the level of perfect health is reset to the perfect health level. In a down step in which the individual drops below the admission line, as long as the new level S(t) is within (0, perfect), the status S(t) is the starting level of the new up phase. In an up step in which the status S(t) exceeds the discharge line, as long as the new level is within (0, perfect),), the status S(t) is the starting level of the new down phase.

It is possible to greatly reduce the amount of simulation required if the longitudinal health status of individuals can be started at the age corresponding to the start of the observation window. A starting status S(A(0)) can be generated as a Uniform random variable over the range of values between the admission line and the line of perfect health. This assumption is not a strict consequence of the model, but for the domain of parameter values used in this study, is a good approximation, since the status sample paths over a lifetime will typically oscillate between two threshold lines several times.

A difficulty is to know how to simulate for each individual the condition of having had a previous hospitalization at the time of the start of the window. Clearly the probability of this happening increases sigmoidally as the individual ages. The model can self-validate the particular choice of sigmoidal function for this probability, by comparison of the simulated age-specific probability of hospitalization in the window with the specification chosen. A choice that worked was the cdf of the Weibull (4,55) distribution.

Observable Data:

The following observations were used to test the simulation model:

i) Hospital stays average 1-2 weeks in length.

ii) Individuals experience an average of three hospital stays in a lifetime, with older individuals having higher frequencies of stays than younger individuals.

iii) Lifetimes that survive early childhood are likely to last an average of about 75 years, but lifetimes over 100 years are very rare. Mortality in the population is about 1 percent per year.

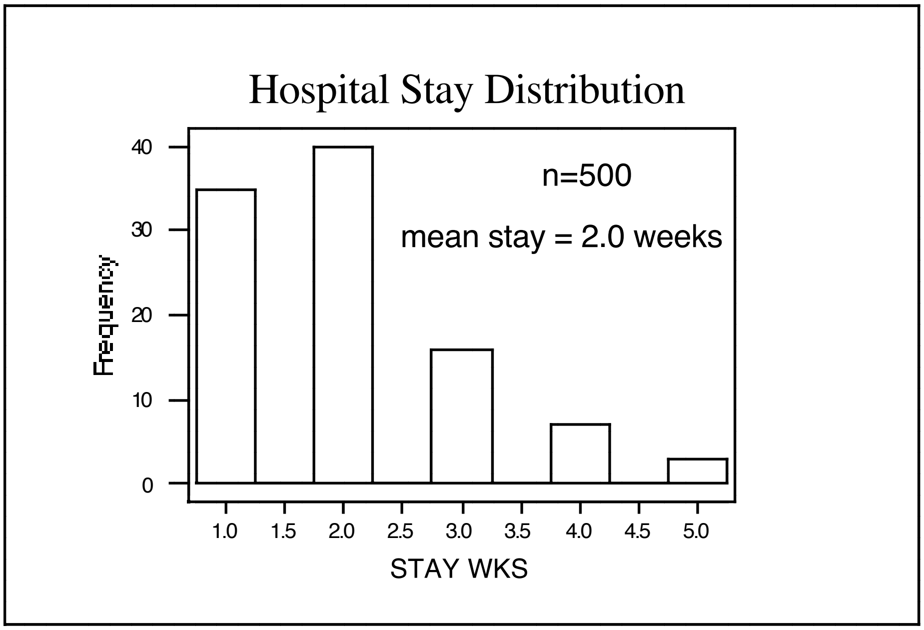
iv) Individuals that have been hospitalized at least once are at higher risk of repeated hospitalization than other individuals.

v) The distribution of hospital stays is uniform across the window. This is simply a check on the homogeneity over the window of the model. A bad model might, for example, produce hospitalizations predominantly in one half of the window.

Some Outcomes:

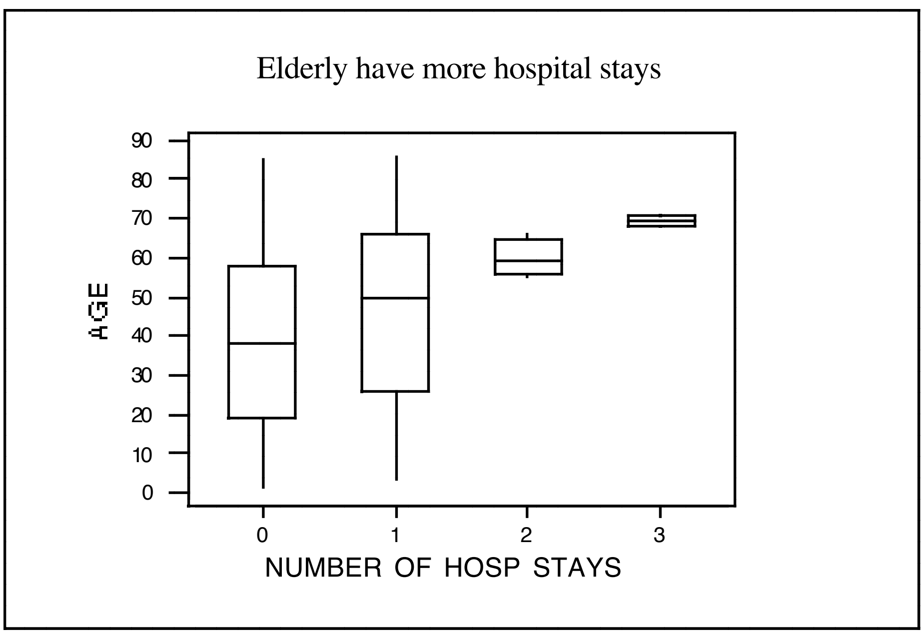
The following results are based on a simulation of a population of 500 individuals. These individuals were captured from a stationary stochastic population at a particular moment in time, and monitored for five years.

i) Hospital stay distribution:



ii) Hospitalization Frequency: The simulation monitors a population for 5 years, or roughly one-fifteenth of a lifetime. For a randomly selected individual from the population at a particular time, there would be about a 20 percent chance that the individual would have a hospital stay during the subsequent five year period. In our simulation of 500 individuals, 18.6% had one or more stays in the five year interval.

Older individuals tended to have more stays in the five year window:



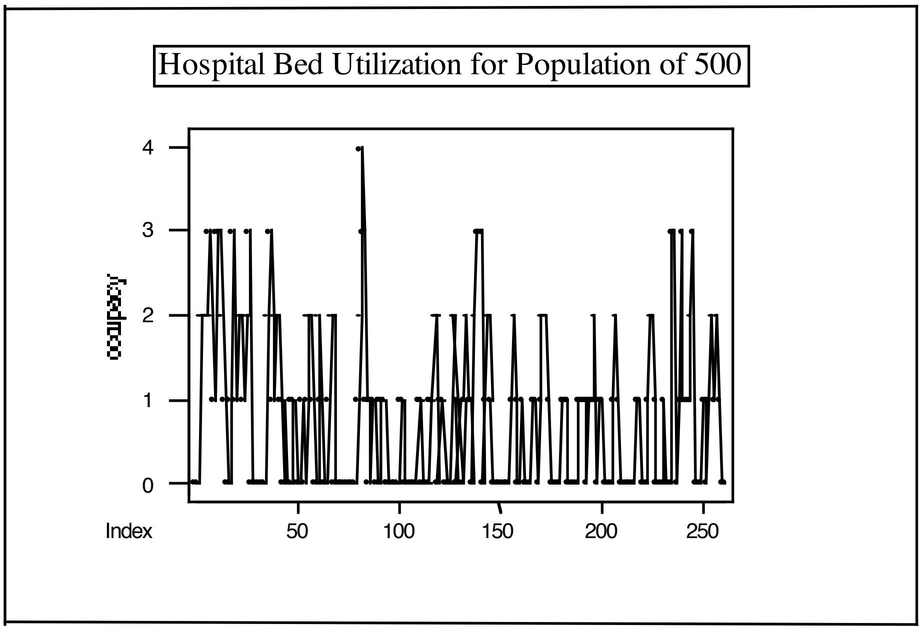
iii) The lifetime distribution for the simulated population is set as Weibull (15, 75). This produces an average lifetime of 72.5 years. However the actual death process in the simulation needs improvement. The number of deaths, 11 out of 500, is a bit higher than the expected number of about 5 out of 500, and the age distribution of the deaths occurring during the observation window is too low, with a mean of about 47 years.

iv) Risk of subsequent hospitalizations:

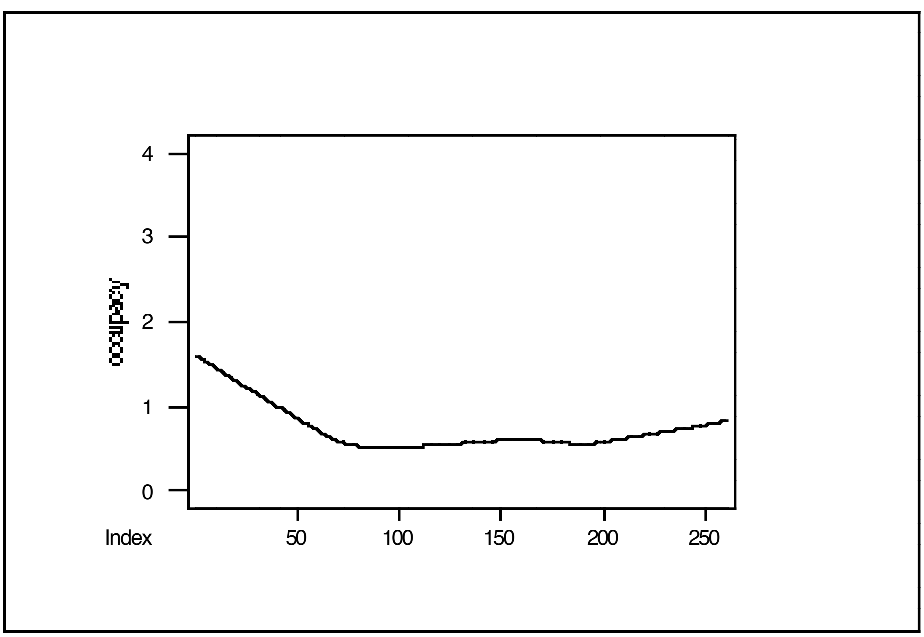
Of those entering the window with no previous hospitalization, 14 percent experienced a hospitalization in the window. Of those entering the window with a previous hospitalization, 26 percent had a subsequent hospitalization in the window. As one would expect, older individuals have more frequent hospitalizations, and this is simply another feature of this fact.

v) Uniformity of hospitalizations over window.

If we record a hospitalization as a square wave with amplitude +1, and sum over all the hospitalizations in the window, the result should be except for random variation, a uniform distribution over the window. The result in our simulation is:



A loess smooth makes the uniformity a little easier to assess:



It is not perfect but fairly uniform.

Note that the unsmoothed graph of occupancy suggests the capacity required to serve this population of 500 individuals. The simulation suggests 3 beds would be adequate, and the smoothed graph shows that the utilization rate of a 3-bed hospital would only be about 33 percent. A 3 bed capacity corresponds to a rate 3/500 or about 6.0 per 1000 population, which is quite close to the actual capacity in Nova Scotia, although the bed utilization is much higher than 33 percent, perhaps 90 percent.

Use of the Model:

The motivation for the model is to provide a way for hospital system administrators to do a preliminary examination of the effects of changes in policy in a risk-free way. System features such as the tendency for a small group of individuals to make frequent use of hospitals can be explored using the model.

A theoretical use of the simulation model is to use a simplified version to relate it to analytical results that are tractable. The analytical results from the simple model can be derived, and the departures from these results, as the model is made more realistic, can be studied using the simulations. Thus a combination of analysis and simulation can make the best use of the full class of models considered.

Conclusions: The model of health status based on a simplistic model does reproduce outcomes that are largely in accord with the real hospital system. While there is still room for improvement in the model, the progress so far suggests that such a model is flexible enough to be calibrated to observable data. Its successful use as a planning and policy tool remains to be demonstrated.

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