Risk management for DC pension plans

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Abstract: Pension Trustees are constrained in their asset class weightings by the reluctance of pension plan members to accept short-term variability in the portfolio return. Trustees must be risk averse in the sense that they must minimize short term losses, and the aversion will generally be greater for asset classes with the larger weightings. Of course the trustees must maximize returns subject to these short-term constraints. The formulation of this problem in mathematical terms is made difficult by the complexity of defining aversion to variability. In this paper we select a utility function that seems to capture both the aversion to short term variability in an asset class as well as its dependence on the weighting of that asset class in the total portfolio. It turns out that the optimization of the portfolio mix is tractable with this utility definition, and Canadian data is used to illustrate the procedure. The commonly used Markowitz efficient frontier requires the investor to state their "risk aversion" by specifying tolerable variability in the total portfolio. But our method produces an optimal mix which takes account of the whole distribution of returns in each asset class, and the correlation of returns among classes, in the maximization of utility. This approach should be useful for pension trustees. For Defined Benefit trustees, the inherent aversion to variability is a natural feature of cash flow management. For Defined Contribution trustees, who are investing for the long term, short-term variability would be less of a concern were it not for the traditions of balanced investing and the possible liability presented by short-term losses.

Environment

In this paper we are concerned with a Defined Contribution pension plan where the Trustees are responsible for the investment decisions on behalf of the members. Although members can select funds from a short list, 85% of members choose to leave their assets in the default fund managed by the trustees. The Trustees are responsible to the members. We are aware of other jurisdictions where individual members must make their own investment choices. Our analysis may be relevant to these jurisdictions as well, but only indirectly.

Investment Objective

Trustees seek to maximize the future value of the contributions to the pension plan by members and employers, subject to practical constraints imposed by the members' attitude towards variability of returns. The future value of a fund is stochastic. It is useful to simulate the range of possible values in future years, but the main aim of this work is

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to show how a single measure of the investment performance can be developed that incorporates the members' attitude to the variability of future returns.

Portfolio Growth

Consider a portfolio of assets A at the start of a period. The assets in the i^{th} sector at the start are given by

$$A_i = p_i A$$

with the proportions satisfying

$$\Sigma p_i = 1$$

If there are no external cash flows then the assets in the i^{th} sector at end the period are given by

$$B_i = A_i (1+ r_i)$$

where r_i is the rate of return for that period.

Total assets at end of the period is

$$B = \Sigma B_i = \Sigma A_i (1+ r_i) = \Sigma p_i A (1+ r_i) = A (1 + \Sigma p_i r_i)$$

so the mean rate of return for the whole portfolio is given by

$$r = \Sigma p_i r_i$$

If the rate of return is not the same for all sectors, then the sectoral proportions of the portfolio change during the period under consideration.

If the portfolio is rebalanced frequently to maintain constant proportions in the various sectors, then a more satisfactory model for the portfolio growth is obtained by replacing the rate of return r with the force of return μ (often referred to as the instantaneous rate of return) where

$$1+ r = \exp(\mu) = \lim_{n\to\infty}(1+\mu/n)^n$$

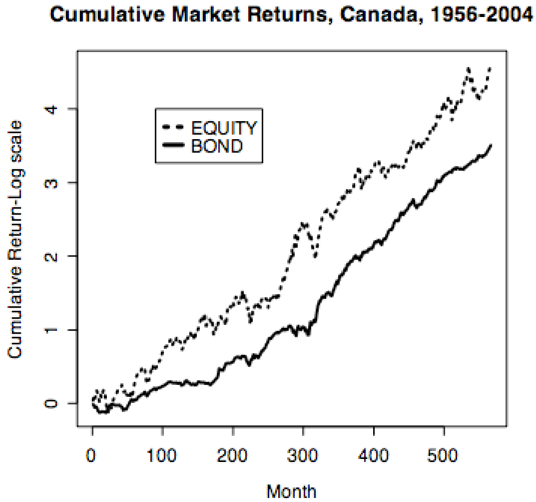
Then working with very small periods between rebalancing the mean force of return for the whole portfolio is given by

$$\mu = \Sigma p_i \mu_i$$

It should be noted that whenever the portfolio is rebalanced, funds are transferred from the sectors with high growth rates in the prior period to those sectors with low growth rates in the same period. The pattern of growth actually experienced may not match the long-term expectation based on sectoral-specific rates of return. The price for maintaining the stability of a fixed mix will be a tendency to lower returns. Nevertheless, we assume a fixed mix in this analysis because of its relevance to current pension practice traditions.

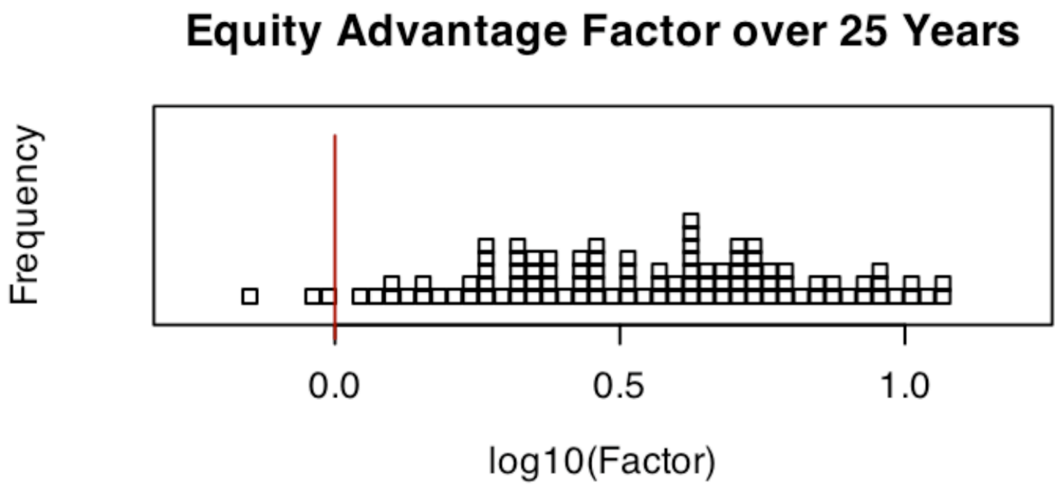
Understanding "Risk": A Simulation of the Canadian Markets.

The Figure below shows the experience of almost 50 years of Canadian markets. The Equity index is the S&P/TSX Total Return Index and the Bond Index is the Scotia Universe Bond Total Return Index from 1980-2004, and the Scotia Capital Mid-term Bond Index for 1956-1979. Both the bond and equity indexes are based on Canadian markets. Although Canadian equities have underperformed US markets, they still have outperformed bonds.



In spite of some nearly catastrophic drops in the equity market, (1972 – oil, 1981-inflation, 1987-computerized trading, 2001 – 9/11), equities are far ahead of bonds over the period. This is the case for any duration of 25 years or more during the period. It is clear that the bond component of the mix reduces the return over these periods. A reasonable conclusion from this graph is that, for periods of 25 years or more, equities outperform bonds, and the greater short-term variability of equities is of little consequence in this comparison. In fact, the long-term investor has almost no risk of underperforming bonds with a portfolio of 100% equities. For the long-term investor, short-term variability does not measure risk. In this context, the "efficient frontier" showing the trade-off between portfolio returns and short-term variability has little relevance to the investor. It is likely that other stable markets would confirm this empirical result.

As another check on the superiority of equities over bonds for pension investing, the Canadian market was simulated by matching a slightly upward drifting random walk model calibrated to match the stochastic characteristics of the real data shown above. The figure below shows a typical result from 100 simulations of a 25 year period:



In a larger simulation, in only 3 percent of the simulations did bonds outperform equities over the 25 year period, and in these unusual cases the bond advantage was slight. The advantage of equities over bonds in the short term is quite subtle, but its long term effect is dramatic. In the simulation we can see that a ratio of exp(.5) = 1.64 would be fairly typical, so it is reasonable to anticipate an annuitized return rate for equities that is 64% greater than for bonds. Put another way, if bonds produced 6 percent per annum, equities might be expected to produce 10 percent per annum, annuitized over 25 years. The simulation used the following parameters for daily changes in index values:

1. The probability of a positive step in one day is .544 for bonds but .547 for equities.
2. The variability of step size for bonds is a bit less with a standard deviation .3% while for equities it is about .5%. Daily changes in Bonds are skewed left whereas for Equities they are symmetrical. Change distributions are exponential except for upward moves for bonds, which is gamma with shape parameter 2.

These parameters do reproduce the general characteristics of the real data from the Canadian market. The simulation suggests the unsurprising result that, in the Canadian market, the consistent superiority of equities over bonds over 25 year periods is a feature of this market likely to persist into the future. Note that it is not merely the mean returns of equities that exceed the returns from bonds over this long term: it is almost the whole distribution. Even though equities have a greater variability of returns, the equity returns are so much greater than for bonds that the equities almost always produce the better return in the time span considered. This is what the figure above demonstrates, and larger scale simulations confirm.

In our environment, defined contribution pension trustees are responsible for the long-term growth of the capital they manage. The contributions are put into the market over a 25-35 year period, and would usually be taken out of the market over a 10-20 year period. This would suggest that a portfolio of 100% equities would provide the best return. Yet the DC Pension Trustee is not likely to choose this apparently obvious strategy of directing member's contributions to 100% equities. The reason has to do with plan member psychology and the traditions of defined benefit (DB) pension plans.

Pension fund members tend to think that gains in market value are expected every year and that losses reflect mismanagement. This attitude is reinforced by the fact that the tradition in pension management is to reduce annual variability by including a sizable proportion of the fund in low-variability asset classes such as bonds. If a high-equity fund were to have a negative year, the low-equity funds would likely do much better in the same year. Only in the longer term would the high-equity fund be proven to be superior. A court might be persuaded that certain individuals do not have the long term to wait. Thus, in spite of the fact that the fund must be managed for the benefit of the whole group, and not any particular subset of members, it may be necessary for trustees to reduce short-term variability in the defined contribution pension fund, to avoid legal challenges. When trustees are elected, re-election could also depend on low short-term variability, so that trustees may also be constrained by members' short-term expectations.

Moderation of short-term variability is very important for the survival of a defined benefit pension plan. DB trustees have to worry about the possibility that at some future time, assets may fall below liabilities. Short-term variability is also important to fund managers of defined contribution (DC) plans whose remuneration depends on the amount of assets under management. But how should the DC pension trustee react to short-term variability? The main reasons for a DC pension trustee to recommend bonds in the portfolio is to maintain the vote of members and to avoid legal liability. These reasons explain why, even for the DC pension trustee, it is advisable to use a utility function which allows for a discounting of average returns that have a high short-term variability. This motivates the remainder of the paper, in which an appropriate utility function is proposed. A procedure to determine the optimal mix of asset classes for maximizing expected utility is described. We describe the results of supplying the procedure to past data from Canadian markets over the last 20 years. That is followed by a discussion of how to apply the technique to future years.

Methodology

Our method is based on a plausible and tractable model for a utility function. With this model, the analysis reduces quickly to standard statistical methods. We provide just a short overview, since the detail is covered in a wide selection of textbooks.

Assume the investor is risk averse, and wishes to give greater weight to the downside variability than the upside. Assume the investor uses an exponential utility function for this purpose, where his utility for a fixed amount of capital x is described by the formula

$$u(x, R) = 1 - \exp(-x/R) \qquad \text{for } R > 0.$$

The parameter R is known as the "risk tolerance" and obviously the same dimension as that of x. In our application, it is related to the total amount to be invested by the Pension Fund Trustees.

If Y is variable amount, then it becomes useful to consider the investor's expected utility.

$$E(u(Y, R)) = 1 - E[\exp(- Y/R)]$$

Since u(x, R) is a 1-1 function of x, we can equivalently work with the "risk-adjusted value of portfolio Y based on risk tolerance R", rav(Y,R), is defined as the unique solution z of

$$1-\exp(-z/R) = 1-E[\exp(-Y/R)]$$

It is easily seen that this unique solution is

$$z \equiv \mathrm{rav}(Y, R) = - R \log(E[\exp(-Y/R)])$$

rav(Y, R) is the exponential premium principle. Buhlmann (1980)

Two properties of the risk-adjusted value that are of special interest are:

(a) <u>Inequality</u>

$$\mathrm{rav}(Y, R) \leq E[Y] \qquad \text{for risk tolerance } R > 0.$$

This property can be proved using Jensen's inequality for convex functions (refer Feller, Vol 2, p.151). At first glance this property seems to be just re-iterating that the investor is risk averse. However cases can arise where rav(Y, R) < 0 while at the same time E[Y] > 0. The investor interprets this negative risk-adjusted value as a danger signal.

(b) <u>Additivity</u>

If variables are independent, then their risk-adjusted values are additive.

To establish this property, we first note that the risk-adjusted value of Y can be written as

$$\mathrm{rav}(Y, R) = - R\, K_Y(-1/R)$$

where

$$K_Y(t) \quad = \log(E[\exp(Yt)])$$

is the cumulant generating function (c.g.f.) of Y.

It is well known that if Z = X + Y where X and Y are independent that

$$K_Z(t) = K_X(t) + K_Y(t)$$

It is now straightforward to show that the desired property holds. This is useful in practical applications.

Portfolio mixtures

If Z = pX where p is a constant multiple then

$$K_Z(t) = K_X(pt)$$

This property can be established from the definition of cumulants of the distribution of a random variable. It follows that if Z = pX + qY where X and Y are independent variables, and p and q are constants, then

$$K_Z(t) = K_X(pt) + K_Y(qt)$$

If X and Y are dependent variables, as is usually the case in practice when X and Y represent different asset classes, then this exact relationship fails to hold, and additional terms are required to restore equality. The adjustment for the second cumulant involves the correlation between the two variables, but this alone will usually be insufficient to account for the higher order cumulants.

One approximate model of interest is the following:

If Z = pX + qY where X and Y are dependent variables, and p and q are constants, then the c.g.f. of Z may be written in the form:

$$K_Z(t) \quad = K_X(pt) + K_Y(qt) + \rho_{XY}\, c_X\, c_Y\, K_X(pt)\, K_Y(qt)$$

where

c_X is coefficient of variation for X,

ρ_{XY} is the correlation between X and Y components.

This form is motivated by its exact validity for independent or perfectly correlated random variables. The generalisation of this model to a mixture with more than two sectors is straightforward, and might be called a copula. It involves all combinations in pairs of prime sectors to take into account their pair-wise correlation.

If $Z = \Sigma\, p_i X_i$ then

$$K_Z(t) \quad = \Sigma_i\, K_{Xi}\,(p_i t) + \Sigma_{i\neq j}\, \Sigma_j\, \rho_{XiXj}\, c_{Xi}\, c_{Xj}\, K_{Xi}(p_i\, t)\, K_{xj}(p_j\, t)$$

where

p_i is proportion of the i^{th} component in the mix,

c_{Xi} is coefficient of variation for the i^{th} component,

ρ_{XiXj} is the correlation between the i^{th} and j^{th} components.

Process for determining the risk-adjusted value of the rate of return of a fund

1. Determine the risk tolerance of the fund.

2. Determine the empirical cumulants the rate of return of the asset classes from past data.

3. Evaluate the risk-adjusted value of the rate of return for each asset class, and check the relative variability in various asset classes. (The difference between the risk-adjusted value and the expected value should reflect our prior knowledge – if the past data is from an unusual time period, we may need to make a manual adjustment.)

4. Add the cumulants of the rate of return for the individual asset classes, weighted in proportion to the fund invested in that class, to obtain a first estimate of the cumulants of the total.

5. Modify the second cumulant of the total to allow for pair-wise correlations between items. Use a copula to generate the adjustment for the higher order cumulants. (Cumulants higher than the fourth are usually ignored, with little effect on the result.)

6. Evaluate the risk-adjusted value of the rate of return for the total fund.

<u>Application to portfolio selection</u>

The classical model for portfolio selection was introduced by Markowitz (1952). The problem is to find the portfolio mix that maximises the return for a given risk. It is assumed that the proportions of the mix are to be held constant over time, so that it is appropriate to use forces of return (and not rates). The Quadratic Programming (QP) formulation of this investment problem is:

maximise	$\mu = \sum p_i \mu_i$	(force of return of mix)
subject to	$v = \sigma^2 = \sum \sum p_i p_j \rho_{ij} \sigma_i \sigma_j$	(variance of mix as measure of risk)
	$\sum p_i = 1$	(constraint on proportions)
	$p_i \geq 0$	(no short selling)

where

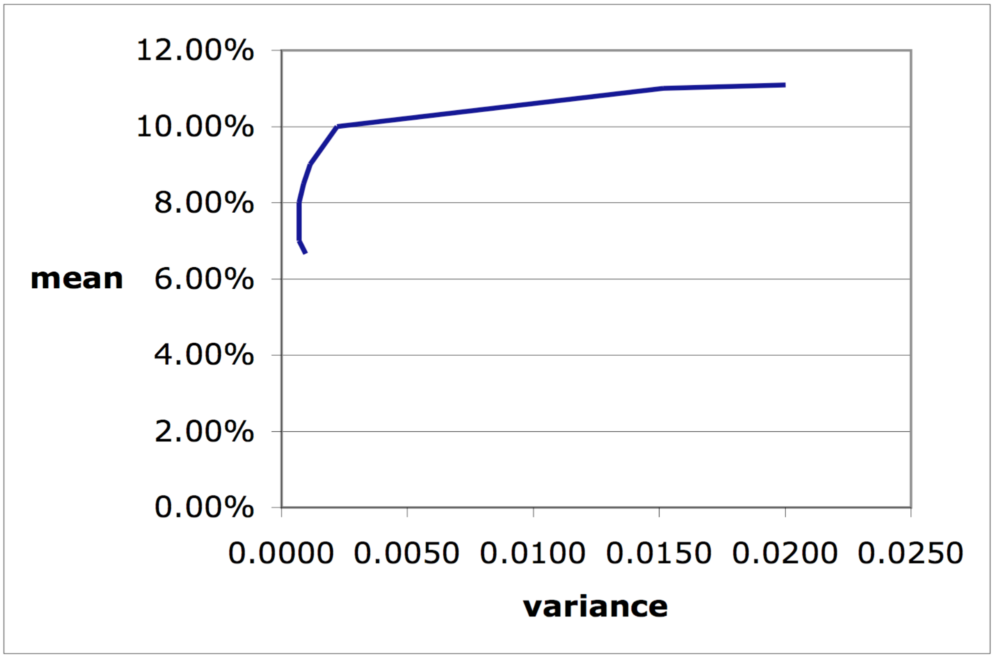
p_i is proportion of the i^{th} component in the mix,
μ_i is mean return for the i^{th} component,
σ_i is standard deviation of return for the i^{th} component,
ρ_{ij} is the correlation of the returns for the i^{th} and j^{th} components,

such that

$-1 \leq \rho_{ij} \leq 1$ if $i \neq j$
$\rho_{ij} = 1$ if $i = j$

The QP can be solved to determine the portfolio mix that corresponds to the efficient frontier. The data used in the following examples is based on historical Canadian data for the period 1984-2004. Details are shown in the Appendix.

The solution to this problem gives rise to the efficient frontier when the results are plotted in the mean-variance plane. This curve is a piecewise parabola. Where does the investor sit? It is usually inferred that the variance is a measure of risk, and the investor has to choose an acceptable level of variance. The efficient frontier curve for this data is shown in the following figure:



Planning for the future

A common application of the classical QP problem, is to use returns that have been determined from the past. Our aim is to plan for the future, when the returns are stochastic. We reformulate the QP problem for this purpose. Furthermore we take the investor's attitude to risk as being specified by his utility function. Our revised formulation of the investment problem is as follows:

maximise	$rav(\mu) = rav(\sum p_i \mu_i)$	(mean return, adjusted for risk)
	$\sum p_i = 1$	(constraint on proportions)
	$p_i \geq 0$	(no short selling)

where

p_i is the proportion of the i^{th} component in the mix,
μ_i is mean return for the i^{th} component,
$rav(\mu)$ is the risk-adjusted value of a portfolio mix μ.

The variance of the portfolio

$$v = \sum \sum p_i p_j \rho_{ij} \sigma_i \sigma$$

where

σ_i is standard deviation of return for the i^{th} component,

ρ_{ij} is the correlation of the returns for the i^{th} and j^{th} components,

can still be estimated, but we no longer use variance of mix as the measure of risk.

Note that we do not assume that the investment returns have a multivariate Normal distribution. The determination of the risk-adjusted values must take into account the higher moments of the distributions.

Risk tolerance in the portfolio selection problem

The trustees of a DC pension fund are responsible for the investment of all contributions received from the members. If we include those contributions not yet invested in the market, we see that the constraint

$$\sum p_i R = R$$

can be re-scaled as

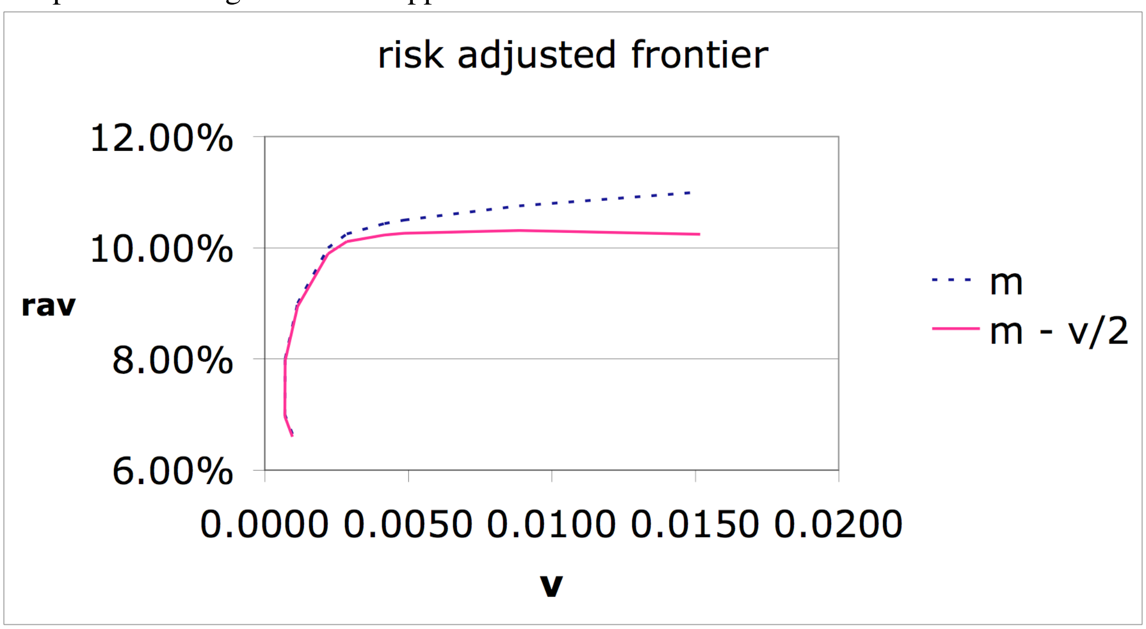
$$\sum p_i = 1$$

Thus, for this problem, the risk tolerance, R, of the investor is

$$R = 1$$

Risk-adjusted frontier

The risk-adjusted frontier can still be plotted in the return-variance plane. The computations are given in the Appendix.



The curve of the risk-adjusted frontier has a unique maximum. The investor's optimum choice is to sit at this maximum. This value maximizes the investors utility.

Solution to Investor's Problem

Maximising the risk-adjusted value, using historical data with R=1, we obtained the following solution.

Cash	Bonds	Cdn Equities	US Equities	Foreign Equities	Real Estate	rav	mean	var
0.0000	0.1939	0.0000	0.6315	0.1746	0.0000	10.26%	10.75%	0.0097

Is this answer reasonable? The mix has a high proportion of equities, but the inclusion of bonds is indicative of a risk averse investor. To test the sensitivity of the result we replaced the historical correlations with a set based on future scenarios, with no adjustment to any of the historical univariate distributions. This led to the following solution.

Cash	Bonds	Cdn Equities	US Equities	Foreign Equities	Real Estate	rav	mean	var
0.0000	0.2939	0.0000	0.7061	0.0000	0.0000	10.17%	10.73%	0.0112

The future scenario of higher correlation between US and Foreign equities has led to the latter being dropped from the optimal mix.

An approximation to the risk-adjusted value

The risk-adjusted value of a random variable Y given by

$$\begin{aligned} \text{rav}(Y, R) &= -R \log(E[\exp(- Y/R)]) \\ &= -R\, K_Y(-1/R) \\ &= k_1 - k_2 /(2\, R) + k_3 /(6\, R^2) - k_4 /(24\, R^3) + \ldots\ldots \end{aligned}$$

A useful approximation is

$$\text{rav}(Y, R) \approx k_1 - k_2 /(2\, R)$$

but this just leads us back to the QP problem.

In particular

$$\text{rav}(\mu, 1) \approx \text{mean - variance / 2}$$

We note that given data on the efficient frontier obtained from the QP problem it is quite easy to obtain mean - variance / 2. Thus in practice it is possible to obtain good approximations to the optimum mix for the risk averse investor without recourse to the c.g.f. or risk averse values. This approximation can enable the use of readily available software, but it should not be taken to imply that higher moments are unimportant.

Conclusions

We have provided a method for selecting an optimal mix of asset classes in a pension portfolio, assuming a certain utility function to describe the investor's attitude to risk. While other utility functions might be proposed, the intractability of the optimal portfolio requires further research before they can be used in the same way. Our utility function does have the merit of providing an example of the risk-averse investor. Of course "risk" in the sense used most commonly is short-term risk. We have argued that it is practical considerations which lead us to use this approach for the long-term investor. It may be that as pension plan members become more knowledgeable about the difference between short-term and long-term investment strategies, that the "optimal" mix of asset classes will change toward a higher proportion of equities.

References

Markowitz,H. (1952) “Portfolio Selection,” Journal of Finance 7: 77-91.
Microsoft® Excel 2004, Microsoft Corporation, Redmond, WA.
Buhlmann, H, (1980) “An Economic Premium Principle”, ASTIN Bulletin 11, 52-60.

Appendix

The optimizations of the portfolio mix were performed using the Solver add-in for Microsoft Excel(2004).

Historical data

Canadian indices 1984-2004

annual force of return	Cash	Bonds	Cdn Equities	US Equities	Foreign Equities	Real Estate
mean	6.65%	9.87%	8.67%	11.09%	10.48%	8.33%
coeff of variation	0.467	0.620	1.551	1.277	1.887	0.790
skewness	0.368	-0.372	-0.331	-0.360	0.249	-0.976
kurtosis	-0.922	0.581	-0.763	-0.673	0.496	0.467

correlation	Cash	Bonds	Cdn Equities	US Equities	Foreign Equities	Real Estate
Cash	1.000	0.376	-0.137	-0.127	0.111	0.006
Bonds	0.376	1.000	0.223	-0.214	0.282	-0.150
Cdn Equities	-0.137	0.223	1.000	-0.167	0.591	0.014
US Equities	-0.127	-0.214	-0.167	1.000	-0.053	0.184
Foreign Equities	0.111	0.282	0.591	-0.053	1.000	0.125
Real Estate	0.006	-0.150	0.014	0.184	0.125	1.000

cumulants	Cash	Bonds	Cdn Equities	US Equities	Foreign Equities	Real Estate
k1	0.06648	0.09875	0.08669	0.11086	0.10482	0.08327
k2	0.00096	0.00375	0.01808	0.02003	0.03914	0.00433
k3	0.00001	-0.00009	-0.00081	-0.00102	0.00192	-0.00028
k4	0.00000	0.00001	-0.00025	-0.00027	0.00076	0.00001
rav	6.60%	9.69%	7.75%	10.07%	8.55%	8.11%

Forecast Correlations

correlation	Cash	Bonds	Cdn Equities	US Equities	Foreign Equities	Real Estate
Cash	1.000	0.080	0.090	0.100	0.080	0.090
Bonds	0.080	1.000	0.310	0.260	0.220	0.120
Cdn Equities	0.090	0.310	1.000	0.770	0.670	0.480
US Equities	0.100	0.260	0.770	1.000	0.750	0.390
Foreign Equities	0.080	0.220	0.670	0.750	1.000	0.330
Real Estate	0.090	0.120	0.480	0.390	0.330	1.000