Stat 280 Test (Ch 1-7) Feb. 10, 1998

Attempt all the questions. You have 50 minutes to earn 50 marks. Leave numerical answers in fractional or unevaluated form if easier.

1. (15 marks) Discrete random variables X and Y have the following joint distribution:

P(X=1, Y=1) = .1
P(X=1, Y=2) = .2
P(X=1, Y=3) = .1
P(X=2, Y=1) = .2
P(X=2, Y=2) = .3
P(X=2, Y=3) = .1

a) Are X and Y independent? Explain or justify.
b) Compute E(X|Y=1)
c) Compute E(X)

1.Ans. a) No because P(Y=i|X=1) is not = P(Y=i|X=2) i=1,2,3.
b) 1*.1/.3+2*.2/.3 = 5/3
c) 1.*(.1+.2+.1) + 2*(.2+.3+.1) = 1.6

2. (10 marks) Two coins have P(Head)=.4 and P(Head)=.6 respectively. One of these coins is selected at random and tossed 5 times, and the outcome is 5 heads. What is the probability that the coin with P(Head)=.6 was selected?

2.Ans Let C be the event that the P(Head)=.6 coin is selected.
P(C|5 heads) = P(5 heads|C)P(C)/P(5heads)
= $.6^5$ (1/2) / [$.6^5$ (1/2)+ $.4^5$ (1/2)] = 1/(1+$(2/3)^5$) =.88 approx.

3. (15 marks) Let X_1, X_2, X_3, …. X_n, be IID random variables such that P(X_i=a) = p and P(X_i=-a) = 1-p. Let S(n,a,p) = $\sum_{i=1}^{n} X_i$.
a) Find the mean and variance of S(n,a,p)
b) Find P[S(5,1,0.5)=3]

3. Ans. a) X_i has mean (pa+(1-p)(-a))= 2pa-a and variance
[$(a-2pa+a)^2$p+ $(-a-2pa+a)^2$(1-p)] = $4a^2$p(1-p)
So mean of S(n,a,p)=n(2pa-a) and var of S(n,a,p)=n[$4a^2$p(1-p)]
b) This symmetric random walk of 5 steps is 3 iff Bin(5,1/2) has 4 forward steps in 5 which has probability 5C4*$(1/2)^5$ = 5/32

4. (10 marks) A manufacturing process produces CDs that are either OK(O), fixable(F), or unfixable(U). One CD is selected from each of 10 batches and inspected. The sequence of outcomes is noted as the batches are sampled, and a sequence like O O F U O O F O O O is observed.
a) How many distinguishable sequences will have 1 U, 2 Fs and 7 Os?
b) If P(U)=.1 and P(F)=.2 and P(O)=.7, what is the probability of 1U, 2Fs and 7Os?

4.Ans.
a) 10!/(2!7!) = 360
b) 360*.1*$.2^2$*$.7^7$ = .12 approx

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