

List of Figures

1.1	A chart (χ, U) and projection mappings.	2
1.2	Two charts in M and a coordinate transformation.	3
1.3	The polar coordinate chart.	4
1.4	Spherical polar coordinates.	5
1.5	Tangent vector in \mathbb{E}_3 and \mathbb{R}^3	6
1.6	A parametrized curve γ into M	12
1.7	Reparametrization of a curve.	13
1.8	The Jacobian mapping of tangent vectors.	23
1.9	A vector field $\vec{U}(x)$ along an integral curve $\xi(\cdot, x)$	36
1.10	A classification chart for manifolds endowed with metric.	65
1.11	Parallel propagation of a vector along a curve.	75
1.12	Parallel transport along a closed curve.	76
1.13	Parallel transport along closed curves on several manifolds	76
1.14	Two-dimensional surface generated by geodesics.	79
1.15	Geodesic deviation between two neighboring longitudes.	81
1.16	A circular helix in \mathbb{R}^3	83
1.17	A two-dimensional surface Σ_2 embedded in \mathbb{R}^3	89
1.18	A smooth surface of revolution.	92
1.19	The image Σ_{N-1} of a parametrized hypersurface ξ	94
1.20	Coordinate transformation and reparametrization of hypersurface ξ	95
1.21	Change of normal vector due to the extrinsic curvature.	99
2.1	A tangent vector \vec{v}_{p_0} in M_4 and its image \vec{v}_{x_0} in \mathbb{R}^4	106
2.2	Null cone \mathcal{N}_{x_0} with vertex at x_0	108
2.3	A Lorentz transformation inducing a mapping between two coordinate planes.	109
2.4	Images Γ_S, Γ_T , and Γ_N of a spacelike, timelike, and a null curve.	112
2.5	The three-dimensional hyper-hyperboloid representing the 4-velocity constraint.	115
2.6	A world tube and a curve representing a fluid stream line.	119
2.7	A doubly-sliced world tube of an extended body.	121
2.8	Mapping of a rectangular coordinate grid into a curvilinear grid in the space-time manifold.	128
2.9	A coordinate transformation mapping half-lines \widehat{L}_+ and \widehat{L}_- into half-lines $\widehat{\widehat{L}}_+$ and $\widehat{\widehat{L}}_-$	130

2.10	Three massive particles falling freely in space under Earth's gravity. . .	140
2.11	Space and time trajectories of two geodesic particles freely falling to- wards the Earth.	141
2.12	Qualitative representation of a swarm of particles moving under the influence of a gravitational field.	142
2.13	Fig. a) shows the parallel transport along a non-geodesic curve. Fig. b) depicts the F-W transport along the same curve.	149
2.14	Measurement of a spacelike separation along the image Γ	151
2.15	A material world tube in the domain $D_{(b)}$	166
2.16	Analytic extension of solutions from the original domain $D_{(e)}$ into \hat{D}	170
2.17	Five two-dimensional surfaces with some peculiarities.	170
2.18	Fig. a) shows a material world tube. Fig. b) shows the continuous $\dot{\mathbf{U}}$ field over Σ	188
2.19	A doubly-sliced world tube of an isolated, extended material body. . .	191
2.20	Domain $D := \mathbf{D}_{(0)} \times (0, t_1) \subset \mathbb{R}^4$ for the initial value problem.	208
3.1	Two-dimensional submanifold M_2 of the Schwarzschild space-time. . .	241
3.2	Rosette motion of a planet and the perihelion shift.	247
3.3	The deflection of light around the Sun.	249
3.4	Two t -coordinate lines endowed with ideal clocks.	250
3.5	Qualitative representation of a spherical body inside a concentric shell. . .	259
3.6	A convex domain D in a two-dimensional coordinate plane.	270
4.1	The two-dimensional and the corresponding axially symmetric three- dimensional domain.	290
4.2	Two axially symmetric bodies in "Euclidean coordinate spaces".	293
4.3	A massive, charged particle at $\mathbf{x}_{(1)}$ and a point \mathbf{x} in the extended body. . .	320
5.1	Qualitative picture depicting two mappings from the Lemaître chart. . .	367
5.2	The graph of the semi-cubical parabola $(\hat{r})^3 = (\hat{v})^2$	368
5.3	The mapping X and its restrictions $X _{\cdot}$	374
5.4	The graph of the Lambert W-function.	375
5.5	Four domains covered by the doubly-null, $u - v$ coordinate chart. . .	377
5.6	The maximal extension of the Schwarzschild chart.	378
5.7	Intersection of two surfaces of revolution in the maximally extended Schwarzschild universe.	382
5.8	Eddington-Finkelstein coordinates (\hat{u}, \hat{v}) describing a black hole.	383
5.9	Qualitative graph of $M(r)$	385
5.10	Collapse of a dust ball into a black hole in a Tolman-Bondi-Lemaître chart.	387
5.11	Collapse of a dust ball into a black hole in Kruskal-Szekeres coordinates. . .	388
5.12	Qualitative representation of a collapsing spherically symmetric star in three instants.	389
5.13	Boundary of the collapsing surface and the (absolute) event horizon. . .	390
5.14	Various profile curves representing horizons in the submanifold $\varphi =$ $\pi/2$, $t = \text{const}$ in the Kerr space-time.	402

5.15	Locations of horizons, ergosphere, ring singularity etc. in the Kerr-submanifold $x^4 = \text{const.}$	404
5.16	The region of validity for the metric in (5.100iii) and (5.99).	410
5.17	The region of validity for the metric in (5.101).	411
5.18	The submanifold M_2^* and its two coordinate charts.	412
5.19	The maximally extended Kerr submanifold \widetilde{M}_2^*	413
5.20	Qualitative representation of <i>an exotic black hole</i> in the T -domain and the Kruskal-Szekeres chart.	423
5.21	Collapse into an exotic black hole depicted by four coordinate charts.	430
5.22	Qualitative graphs of $y = [\theta(s)]^{-1}$ and the straight line $y = \Omega(s) := y_0 + (1/3) \cdot (s - s_0)$	433
6.1	Qualitative graphs for the “radius of the universe” as a function of time in three Friedmann (or standard) models.	440
6.2	Qualitative representation of a submanifold M_2 of <i>the spatially closed space-time</i> M_4	441
6.3	Qualitative graphs of $y = [\theta(s)]^{-1}$ and the straight line $y = y_0 + \frac{1}{3}(s - s_0)$	450
6.4	Comparison of the square of the cosmological scale factor, $a^2(t)$	467
6.5	Qualitative graphs of two functions $\zeta(\beta)$ and $z(\beta)$ corresponding to <i>the particular function</i> $h(\beta) := \varepsilon \cdot \beta^{-1}$	476
6.6	The qualitative graph of the function $\zeta(\beta)$ for $0 < \beta < 1$	477
6.7	Qualitative graphs of a typical function $h(\beta)$ and the curve comprising of minima for the one-parameter family of such functions.	478
6.8	Graphs of evolutions of functions depicting the scale factors.	480
7.1	A tetrad field containing two spacelike and two null vector fields.	484
8.1	A plot of the function in (8.39).	573
8.2	A plot of several solutions to the Emden equation.	576
8.3	The graph of the function $r = \coth x - 1 > 0$	580
8.4	Graphs of the eigenfunctions $U_{(0)}(x)$, $U_{(1)}(x)$ and $U_{(2)}(x)$	584
8.5	Properties of the solution $u_{(0)}(r)$	588
8.6	Qualitative plots of three null cones representing radial, null geodesics.	589
8.7	Properties of the solution $u_{(2)}(r)$	590
A1.1	Two twice-differentiable parametrized curves into \mathbb{R}^N	595
A1.2	The mappings corresponding to a tensor field $\phi^{(r+s)}(x)$	597
A1.3	Representative spacelike hypersurfaces in an ADM decomposition of space-time.	607
A2.1	Classification diagram of p.d.e.s.	612
A2.2	Graphs of non-unique solutions.	627
A5.1	Quantities relevant in the solution of the linearized field equations.	650
A5.2	The + and \times polarizations of gravitational waves.	652
A5.3	Detection parameters of the LIGO and LISA detectors.	653
A6.1	A possible picture for the space-time foam.	655
A6.2	An inter-universe and intra-universe wormhole.	656

A6.3	A cross section of a wormhole profile curve and corresponding surface of revolution.	657
A6.4	A “top-hat” function.	660
A6.5	The expansion of spatial volume elements in a warp-drive space-time.	661
A6.6	The distribution of energy density for a warp-drive space-time. . . .	662
A6.7	Two examples of closed timelike curves.	663
A6.8	The embedding of anti-de Sitter space-time in a five dimensional flat space-time with two dimensions suppressed.	664
A6.9	The light-cone structure about an axis $\varrho = 0$ in the Gödel space-time.	665