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## THE DEMAND FUNCTION AND THE DEMAND CURVE

These notes try to make clear the relationship and the distinction between the concepts of "demand function", "the demand curve" and the "inverse demand function".

When economists first started to think about demand as the relationship between the quantity consumed of a good and its price holding everything else (other prices, income, preferences, etc.) fixed, they put the price (say $p_{1}$ ) on the vertical axis and quantity ( $x_{1}$ ) on the horizontal axis, i.e. expressed price as a function of quantity holding all else fixed. Such graph of the relationship between price and quantity with the price on the vertical axis is know as the demand curve. For every given quantity $x_{1}$ the demand curve shows us what the price of the good should be in order for the consumer to be optimally consuming (choosing) $x_{1}$. Alternatively, using the demand curve we can read from the graph the quantity demanded that would prevail at a given price.

Later on, economists developed the whole consumer theory that we are studying now and defined the demand function as the relationship between the quantity of the good consumed at optimum and prices and income, i.e. $x_{1}\left(p_{1}, p_{2}, m\right)$ is the demand function for good 1 . Notice that in general this is a function of three variables so we cannot plot it on a two dimensional graph. To be able to do that, and to isolate only the effect of the price of the good, $p_{1}$ on the quantity demanded we can hold $m$ and $p_{2}$ as fixed (at some constant values, $\bar{m}$ and $\bar{p}_{2}$ ) and thus obtain a function $x_{1}\left(p_{1}, \bar{p}_{2}, \bar{m}\right)$ where $\bar{m}$ and $\bar{p}_{2}$ are treated as parameters (say like $\alpha$ in the Cobb-Douglas function). This simplified version of the demand function can be plotter on a two dimensional graph with $x_{1}$ on the vertical axis and $p_{1}$ on the horizontal and it is exactly the mirror image of the demand curve if we flip the axes. If we want to have price as a function of quantity (as in the demand curve) we can take the function $x_{1}=x_{1}\left(p_{1}, \bar{p}_{2}, \bar{m}\right)$ and "invert" it to find $p_{1}=p_{1}\left(x_{1}, \bar{m}, \bar{p}_{2}\right)$. This function is called the inverse demand function and its graph is the demand curve.

## Example:

Take the perfect complements demand function for good 1

$$
x_{1}=x_{1}\left(p_{1}, m, p_{2}\right)=\frac{m}{p_{1}+p_{2}}
$$

If we fix $m$ and $p_{2}$ at some constant values, e.g. $m=\bar{m}=10, p_{2}=\bar{p}_{2}=2$ then we get $x_{1}$ just in terms of $p_{1}$ (i.e. the demand function) for these fixed values of $p_{2}$ and income:

$$
\begin{equation*}
x_{1}=x_{1}\left(p_{1}, \bar{p}_{2}, m\right)=\frac{10}{p_{1}+2} \tag{1}
\end{equation*}
$$

which we can plot on a graph with $x_{1}$ on the vertical axis and $p_{1}$ on the horizontal axis but this graph will not be the demand curve since quantity and price are on the "wrong" axes.

If we want to plot the demand curve we need to take the above graph and flip the axes - i.e. express $p_{1}$ as a function of $x_{1}$ holding $m$ and $p_{2}$ fixed. To do that just take the above equation (1) and express $p_{1}$ in terms of $x_{1}$ :

$$
p_{1} x_{1}+2 x_{1}=10
$$

or,

$$
p_{1}=\frac{10}{x_{1}}-2
$$

which is the inverse demand function $p_{1}\left(x_{1}, \bar{m}, \bar{p}_{2}\right)$, the graph of which is the demand curve.

