Practice and Exercises #4

Part 4: Optimization

- **Exercise 1:**

Solve the following problem:

$$\min_{r,f}[7r+30f] \quad s.t. \begin{cases} r \geq 0, \\ f \geq 0, \\ 120r+70f \geq 1000 \\ .25f \geq 1.25 \\ 3r+15f \geq 25 \\ 26r \geq 50 \end{cases}$$

- **Exercise 2:**

Solve the following optimization problem:

$$(P_1) \quad \max_{C,L}[a\ln(C)+(1-a)\ln(T-L)] \quad s.t. \begin{cases} C \leq wL+I \\ T \geq L \\ C \geq 0 \\ L \geq 0 \end{cases}$$

Hints: (i) make sure you study all the cases associated with the KT conditions;
(ii) your solution will depend on the value of the parameter a (you should be able to distinguish two cases.)

- **Exercise 3:**

1. Solve the following optimization problem when $8 < m < 40$:

$$\max_{x,y}[xy+y^2+2x+2y] \quad s.t.\ 6x+10y=m,\ x \geq 0,\ y \geq 0$$

2. What are the solutions for x^* and y^* if $m \leq 8$? What are the solutions if $m \geq 40$?

- **Exercise 4:**

1. Solve the following optimization problem:

$$\max_{S,L}\{100S+250L\} \quad s.t. \begin{cases} S+3L=200 \\ 0.5S+L=80 \\ S \geq 0 \\ L \geq 0 \end{cases}$$

2. Suppose the 2 equality constraints are related to the same constrained resource. And suppose you now have access to an additional unit of this resource. Is it better to use this additional unit to increase 200 by 1? or to increase 80 by 1? Explain.

   Note: to put things into perspective, think about these 2 equality constraints in terms of time constraints when completing a given task. A company's production process requires 2 tasks to be completed (each necessitates some time); the company can hire an additional worker (to somewhat relax one of the time constraint), but needs to decide which task the additional worker should complete.