Bayesian estimation of acoustic aberrations in high intensity focused ultrasound treatment

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June 28, 2016

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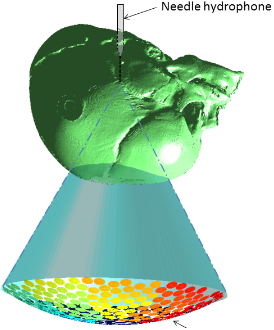
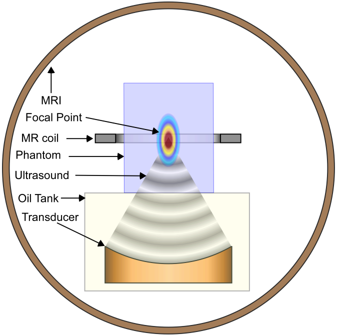
What is HIFU?

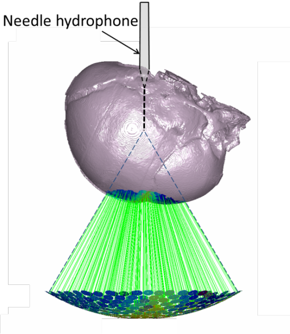
- High intensity focused ultrasound (HIFU).
- A focused beam of acoustic waves converging in a small volume.
- The generated heat ablates diseased tissue.

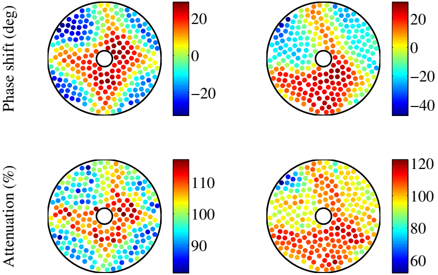




What are the challenges?

- Clinical success in treatment of prostate cancer, liver tumours, uterine fibroids, etc.
- Treatment of brain tumours remains a challenge.
- Strong aberrations due to skull bone → defocused beam.
- Estimate aberrations → compensate phase shift → refocus.



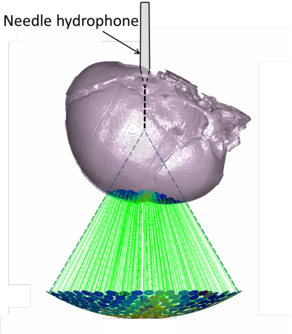


An approximate forward model

The pressure field due to a single piezoelectric element:

$$p(\mathbf{x}) = p_0 \ \exp\left(i\left[\omega t + \frac{\omega}{c_0}|\mathbf{x}|\right]\right) \times \mu \qquad \mu = \zeta \exp(i\phi t).$$

- p_0 signal amplitude.
- ω frequency.
- c_0 speed of sound.
- μ aberration due to tissue
- ζ is attenuation
- ϕ phase shift



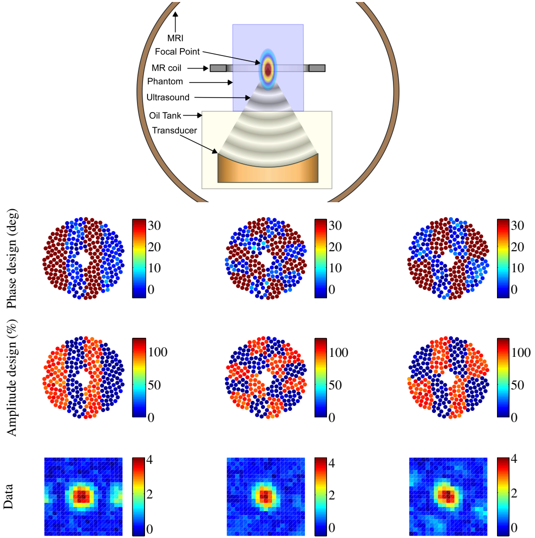
Energy based measurements

- Assemble in to a linear forward map:

$$\mathbf{p} = \mathbf{FZa}$$

- $\mathbf{F}$ free field matrix (Green's function).
- $\mathbf{Z}$ input phase and amplitude (experimental design).
- $\mathbf{a}$ aberration due to tissue.
- Measure the amplitude of pressure

$$\mathbf{d} = \mathrm{diag}(\mathbf{p})\mathbf{p}^*.$$



The inverse problem

- Estimate the vector of aberrations $\mathbf{a}$ given the data $\mathbf{d}$ and matrices $\mathbf{F}$ and $\mathbf{Z}$.
- Use Bayes' rule[1]:

$$\pi(\mathbf{a}|\mathbf{d}) \propto \pi(\mathbf{d}|\mathbf{a})\pi_0(\mathbf{a}).$$

- $\pi(\mathbf{d}|\mathbf{a})$ likelihood.
- $\pi_0(\mathbf{a})$ prior.
- $\pi(\mathbf{a}|\mathbf{d})$ posterior.

[1] J. Kaipio and E. Somersalo. *Statistical and Computational Inverse Problems*. Springer Science and Business Media, New York, 2005.

The likelihood

$$\pi(\mathbf{a}|\mathbf{d}) \propto \pi(\mathbf{d}|\mathbf{a})\pi_0(\mathbf{a}).$$

- Additive noise model

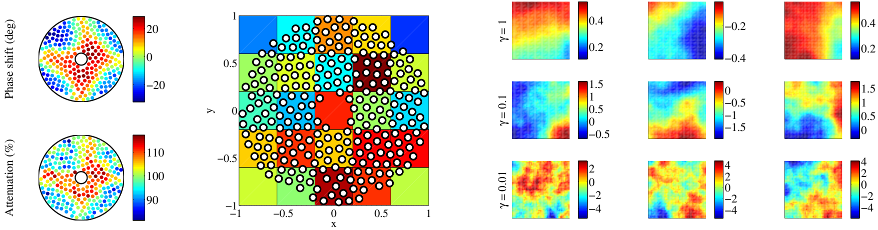
$$\mathcal{G}(\mathbf{a}) := \mathrm{diag}(\mathbf{FZa})(\mathbf{FZa})^* \qquad \mathbf{d} = \mathcal{G}(\mathbf{a}) + \boldsymbol{\epsilon}, \qquad \boldsymbol{\epsilon} \sim \mathcal{N}(0, \boldsymbol{\Sigma}).$$

- Likelihood:

$$\pi(\mathbf{d}|\mathbf{a}) \propto \exp\left(-\frac{1}{2}\left\|\boldsymbol{\Sigma}^{-1/2}(\mathbf{d} - \mathcal{G}(\mathbf{a}))\right\|_2^2\right)$$

Choosing the prior π_0

$$\pi(\mathbf{a}|\mathbf{d}) \propto \pi(\mathbf{d}|\mathbf{a})\pi_0(\mathbf{a}).$$



- Measured aberrations hint at an underlying continuous field.
- Construct $\pi_0(\mathbf{a})$ by pointwise evaluation of a Gaussian random field:

$$\mathcal{N}(0, (\mathcal{I} - \gamma\Delta)^{-2})$$

- $(\mathcal{I} - \gamma\Delta)^{-2}$ biharmonic operator with Neumann boundary condition.

Sampling from π_0

- Pick $\sigma > 0$.
- Discretize $(\mathcal{I} - \Delta)^{-2}$.
- Sample

$$\mathbf{u} \sim \mathcal{N}(0, (\mathcal{I} - \gamma\Delta)^{-2}), \qquad \alpha_1 \sim N(0, \sigma)$$
$$\mathbf{v} \sim \mathcal{N}(0, (\mathcal{I} - \gamma\Delta)^{-2}), \qquad \alpha_2 \sim N(0, \sigma)$$

- Set $\mathbf{a} = \text{diag}(\alpha_1^2 \mathbf{u}) \exp(i\alpha_2^2 \mathbf{v})$.

Sampling from the posterior $\pi(\mathbf{a}|\mathbf{d})$

- Forward problem is nonlinear $\rightarrow$ posterior $\pi(\mathbf{a}|\mathbf{d})$ is not Gaussian.
- Markov Chain Monte Carlo.
- Requires sampling from the prior π_0 and evaluating likelihood $\pi(\mathbf{d}|\mathbf{a})$.

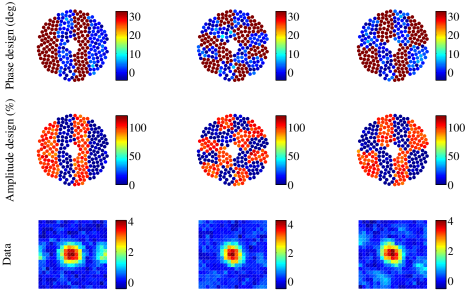
$$\pi(\mathbf{d}|\mathbf{a}) \propto \exp\left(-\frac{1}{2}\left\|\mathbf{\Sigma}^{-1/2}(\mathbf{d}-\mathcal{G}(\mathbf{a}))\right\|_2^2\right).$$

- Differentiable in real arguments, not differentiable in the complex arguments.
- MALA + Random walk block sampler[2] (at every step):
  (1) Fix $\mathbf{v}$ and α_2 and sample $\mathbf{u}$ and α_1 using Metropolis adjusted Langevin algorithm (uses gradient information).
  (2) Fix $\mathbf{u}$ and α_1 and sample $\mathbf{v}$ and α_2 using preconditioned Crank-Nicolson random walk algorithm.

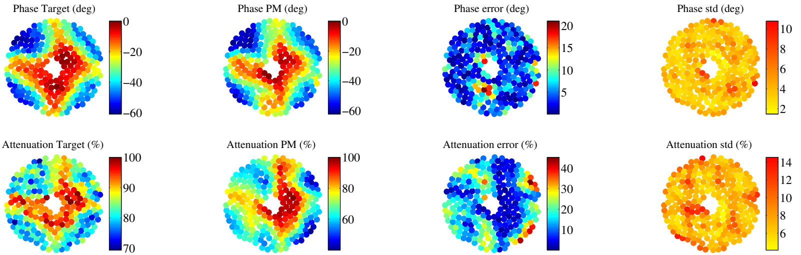
[2] S. L. Cotter et al. "MCMC methods for functions: modifying old algorithms to make them faster". In: *Statistical Science* 28.3 (2013), pp. 424–446.

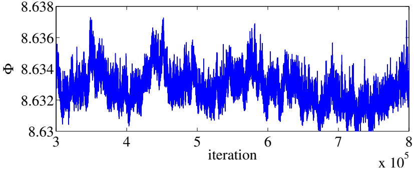
Synthetic experiment

- Philips Sonalleve device with 256 piezoelectric elements.
- 512 unknowns.
- Discretize the continuous field on a 8×8 mesh (64 dimensional parameter space).
- 16 sonication tests.
- 19×19 voxel MRI window for each sonication test.
- $SNR = 5$.
- 3×10^5 burn-in $+ 5 \times 10^5$ steps of MCMC.



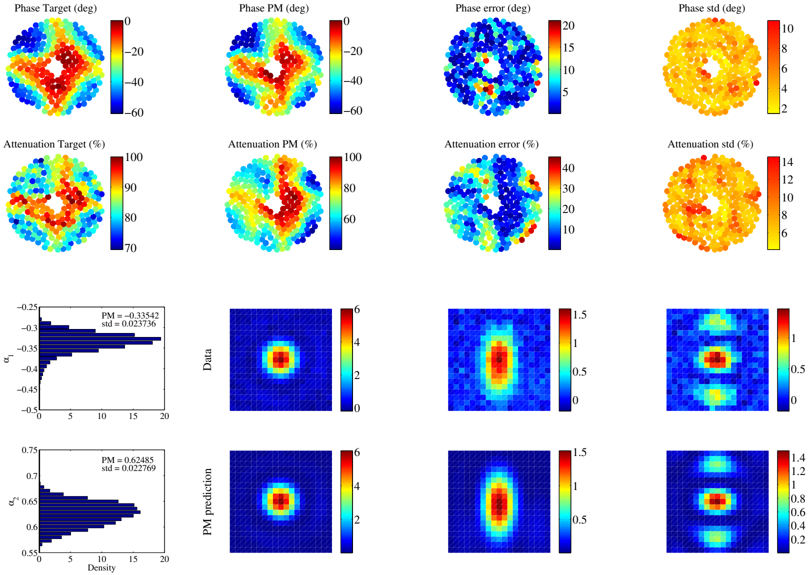
Synthetic experiment



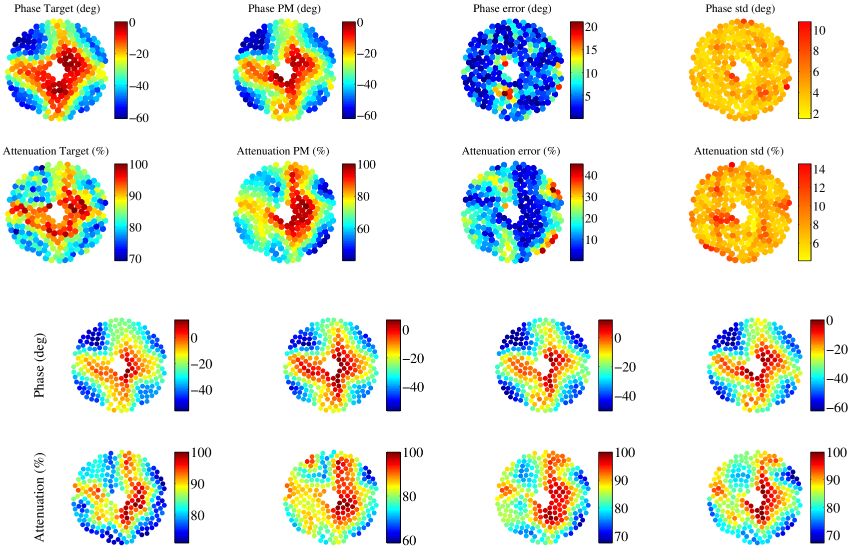


$$\Phi := -\frac{1}{2}\left\|\boldsymbol{\Sigma}^{-1/2}(\mathbf{d}-\mathcal{G}(\mathbf{a}))\right\|_2^2$$

Synthetic experiment



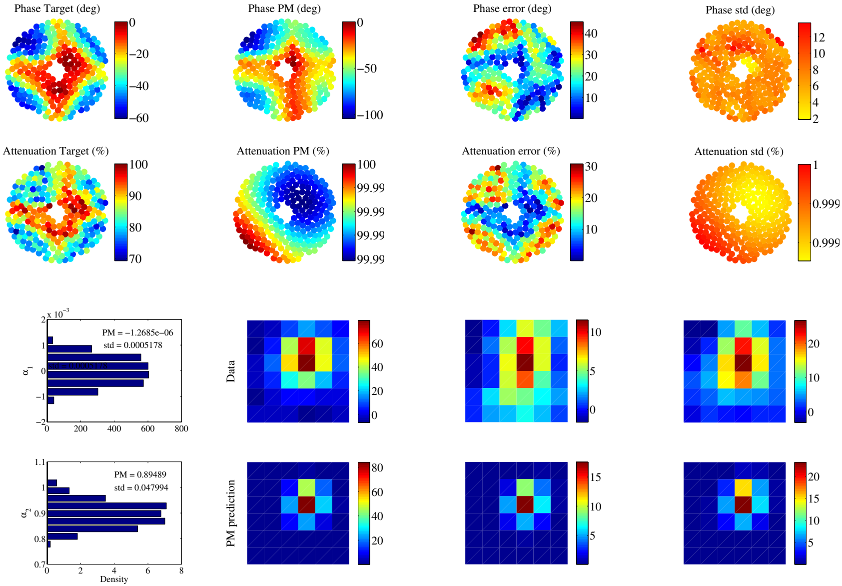
Synthetic experiment



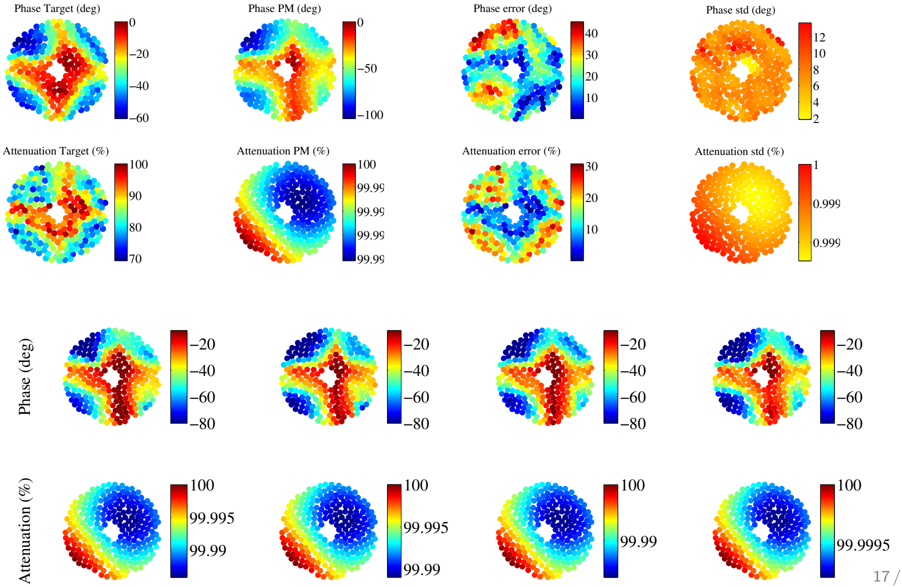
Physical experiment

- The same Philips Sonalleve device.
- Artificial aberrator.
- Discretize the Gaussian field on a 8×8 grid.
- 32 sonication tests.
- 7×7 voxel MRI window for each test.
- 3×10^5 burn-in + 5×10^5 steps of MCMC.

Physical experiment



Physical experiment



Selling points

For the practitioner:

- The "state of the art"[3] uses 256 sonication tests as compared to 32 tests used here.
- Patient spends less time in the MRI machine.
- Save energy.
- flexible design of experiments (choice of matrix $\mathbf{Z}$).

For the mathematician:

- Well-posed inverse problem.
- Estimate uncertainty.
- Compute in 2-4 hours on a laptop.
- Experimental design.

[3] E. Herbert et al. "Energy-based adaptive focusing of waves: application to noninvasive aberration correction of ultrasonic wavefields". In: *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control* 56.11 (2009), pp. 2388–2399.

Challenges and future directions

Challenges:

- Large discrepancy between the model and physical process
- Calibration of $\mathbf{F}$.
- Better data.
- Regularizing the likelihood.
- Better sampling algorithm.

Future direction:

- Phase retrieval techniques[4].
- Matrix completion to estimate the free-field $\mathbf{F}$.
- Design of experiments.

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[4] E. J. Cándes et al. "Phase retrieval via matrix completion". In: *SIAM Review* 57.2 (2015), pp. 225–251.

Acknowledgement

- Fields institute and the organizers of Fields-Mprime Industrial Problem Solving Workshop, August 2014.
- NSERC
- Brain Canada Multi-Investigator Research Initiative
- Focused Ultrasound Foundation

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MCMC performance