

Chapter 5

Real business cycles

5.1 Real business cycles

The most well known paper in the Real Business Cycles (RBC) literature is Kydland and Prescott (1982). That paper introduces both a specific theory of business cycles, and a methodology for testing competing theories of business cycles.

The RBC theory of business cycles has two principles:

1. Money is of little importance in business cycles.
2. Business cycles are created by rational agents responding optimally to real (not nominal) shocks - mostly fluctuations in productivity growth, but also fluctuations in government purchases, import prices, or preferences.

The “RBC” methodology also comes down to two principles:

1. The economy should always be modeled using dynamic general equilibrium models (with rational expectations).
2. The quantitative implications of a proposed model should be taken seriously. In particular, a model’s suitability for describing reality should be evaluated using a quantitative technique known as “calibration”. If the model “fits” the data, its *quantitative* policy implications should be taken seriously.

Romer, writing just a few years ago, treats these two meanings of the term as essentially the same. But since then, the RBC methodology has taken hold in a lot of places - researchers have analyzed “RBC” models with money, with all sorts of market imperfections, etc. In order to reflect this change, researchers are trying to change the name of the methodology to the more accurate (if awkward) “stochastic dynamic general equilibrium macroeconomics.”

5.1.1 Some facts about business cycles

Business cycles are the reason why macroeconomics exists as a field of study, and they’re the primary consideration of many macroeconomists. What characterizes business cycles? The two obvious characteristics are fluctuations in unemployment and output. A few definitions:

- Procyclical: a variable that usually increases in booms, decreases in recessions. For example, productivity is procyclical.
- Countercyclical: a variable that usually decreases in booms, increases in recessions. For example, unemployment is countercyclical.
- Acyclical: a variable that shows no systematic relationship to the business cycle.
- Fiscal policy: the government’s policy for taxes and spending.
- Monetary policy: the government’s policy for how much money to put into the economy.

Before we go into the details of an RBC model, let’s establish some stylized facts about business cycles. Most of these are outlined in the beginning of Chapter 4 in Romer.

1. Labor input varies considerably and procyclically (goes up in booms, down in recessions). Most of this variation is variation in employment rates, though some is in average weekly hours.
2. The capital stock varies little at business cycle frequencies (1-3 years).

3. Productivity growth (as measured by the Solow residual) is procyclical, though not nearly as much as labor input. In other words, most of the output loss in recessions can be traced to unemployment.
4. Wages vary less than productivity, and have low correlation with output.
5. All major expenditure categories are procyclical. Investment in consumer and producer durables is quite volatile, while consumption of nondurables and services varies much less than output. The most volatile expenditure category is inventory investment.

One of the main research programs in macro has been to explain these facts in a consistent manner (using a well-specified equilibrium model). A few major puzzles:

- Why do labor inputs vary? Specifically, if there is cyclical unemployment, what shock causes labor demand to go down? Once labor demand goes down, why is this reflected in unemployment rather than wage cuts?
- Decreasing returns to scale in labor implies countercyclical productivity, but productivity is procyclical. Why?
- Why are recessions so persistent?

Returning to what we said earlier, one of the key puzzles in business cycle theory is procyclical productivity. The RBC theory explains procyclical productivity quite directly - booms are good draws of technology growth recessions are bad draws.

The second puzzle is employment. In the baseline RBC model, employment fluctuations are just intertemporal substitution of leisure. In some time periods, labor is less productive than others. The optimal action for workers is to work more in productive periods, less in unproductive periods. This explains unemployment.

The third puzzle is persistence. In the baseline RBC mode, capital accumulation is the “internal propagation mechanism” - the thing that converts shocks without persistence into highly persistent shocks to output. If the

technology level is below average, output is low, so investment is low, so the next period's capital stock is also below average. So even if the technology level returns to normal next period, output will be below normal.

A fourth observation (not so much a puzzle) is why investment spending is more variable than consumption spending. This is not so hard to explain in the baseline RBC model (or any other model): an agent with the preference to smooth consumption over time will invest in productive periods and eat capital in unproductive periods.

5.1.2 The model

Now let's put this into a model. We'll take the equilibrium growth model that we've already looked at, and add a labor-leisure choice, and a stochastic technology. This produces what's commonly known as the "baseline real business cycle model". The version of the model I'm outlining here is from Chapter 1 in Thomas Cooley's *Frontiers of Business Cycle Research*. The chapter is written by Cooley and Prescott. The model described in Romer is basically the same.

Consumers

The consumer's problem is the same as before, except that he values leisure. He supplies labor $h_t \in [0, 1]$ in period t , just like before, but now he receives utility from leisure. The consumer also has uncertainty over future prices, so he maximizes *expected* utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - h_t) \quad (5.1)$$

subject to the constraints:

$$\begin{aligned} x_t + c_t + b_{t+1} &\leq w_t h_t + r_t k_t + R_t b_t + \pi_t \\ k_{t+1} &\leq (1 - \delta) k_t + x_t \\ k_t &\geq 0 \\ k_0 &\text{ given} \end{aligned}$$

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We assume that the consumer is making all time- t choices $(x_t, c_t, k_{t+1}, b_{t+1}, h_t)$ conditional on time t information (all variables subscripted t and below, plus the interest rate on bonds R_{t+1}).

Notice that we will be modeling fluctuations in employment as a representative consumer varying his hours. Of course, employment fluctuations are a lot “lumpier” than that - this may affect the outcomes we look at.

The firm

We’ve talked about the consumer dealing with stochastic prices. We haven’t discussed the source of uncertainty. That will be on the firm side. The firm is the same as before. However, the production function is subject to random productivity shocks: The firm’s problem is:

$$\max_{K_t, H_t} e^{z_t} F(K_t, H_t) - w_t H_t - r_t K_t \quad (5.2)$$

where F is a neoclassical production function and z_t follows an AR(1) process:

$$z_t = \rho z_{t-1} + \epsilon_t \quad (5.3)$$

where ϵ_t is white noise.

Equilibrium

When variables are stochastic, equilibrium is defined slightly differently.

An equilibrium in this economy is a joint distribution of prices and allocations such that, etc.

5.1.3 Solving the model

The consumer’s Lagrangian is:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - h_t) + \lambda_t ((1 - \delta)k_t + w_t h_t + r_t k_t + \pi_t - c_t - k_{t+1}) \quad (5.4)$$

The first order conditions are:

$$c_t : E_t(\beta^t u_c(c_t, 1 - h_t) - \lambda_t) = 0 \quad (5.5)$$

$$h_t : E_t(-\beta^t u_\ell(c_t, 1 - h_t) + \lambda_t w_t) = 0 \quad (5.6)$$

$$k_{t+1} : E_t(\lambda_{t+1}(1 - \delta + r_{t+1}) - \lambda_t) = 0 \quad (5.7)$$

$$b_{t+1} : E_t(\lambda_{t+1}(R_{t+1}) - \lambda_t) = 0 \quad (5.8)$$

plus the capital accumulation equation and the transversality condition. The appropriate expectations operator is E_t because each of these variables is chosen with the information available at time t . We can eliminate the expectations operator if the variables inside it are in the time t information set. So we have:

$$\lambda_t = \beta^t u_c(c_t, h_t) \quad (5.9)$$

as in the basic equilibrium growth model, and:

$$u_\ell(c_t, 1 - h_t) = u_c(c_t, 1 - h_t)w_t \quad (5.10)$$

This equation shows that the worker equates the marginal utility from a little more leisure to the utility from working an equal amount and getting more consumption. In addition, we have:

$$u_c(c_t, 1 - h_t) = \beta E_t [u_c(c_{t+1}, 1 - h_{t+1})(r_{t+1} + 1 - \delta)] \quad (5.11)$$

which is just the same as Euler equation for the basic growth model, but with the expectations operator.¹ Finally, the bond price is given by:

$$R_{t+1} = E_t(r_{t+1}) + 1 - \delta \quad (5.13)$$

The solution to the firm's problem implies:

$$w_t = e^{z_t} F_H(K_t, H_t) \quad (5.14)$$

$$r_t = e^{z_t} F_K(K_t, H_t) \quad (5.15)$$

¹Remember that

$$E(XY) = Cov(X, Y) + E(X)E(Y)$$

So we can rewrite (5.11) as:

$$u_c(c_t, h_t) = \beta[E_t(u_c(c_{t+1}, h_{t+1}))(E_t(r_{t+1}) + 1 - \delta) + Cov_t(u_c(c_{t+1}, h_{t+1}), r_{t+1})] \quad (5.12)$$

We see that today's decision doesn't just depend on the expected value of future prices but also their covariance with consumption.

5.1.4 Calibration

Kydland and Prescott suggest a way to identify if this model can explain business cycles. Their method is known as calibration. The procedure is:

- Use microeconomic studies or theory to find values for all of the parameters.
- Solve the model numerically, and simulate the economy.
- Compare the moments (standard deviations, correlations, etc) of the simulated economy with those in the actual economy.
- If the moments are matched, success!
- If not, the moments which don't match up suggest areas of potential model improvement.

We're going to do this with the model. First, we need to put parametric functional forms. First, utility:

$$u(c, 1 - h) = \frac{(c_t^{1-\alpha}(1 - h_t)^\alpha)^{1-\chi}}{1 - \chi} \quad (5.16)$$

This is a variation on the CRRA form we've been using. Note that $1/\chi$ is the intertemporal elasticity of substitution. When $\chi = 0$, utility is linear, when $\chi = \infty$, utility is Leontief, and when $\chi = 1$, utility is Cobb-Douglas (logarithmic).

Production is assumed to be Cobb-Douglas:

$$F(K, H) = K^\theta H^{1-\theta} \quad (5.17)$$

Now, we need plausible parameter values for this model. I'll use the values from Cooley and Prescott (1995). Their model is identical to the one here, except that they allow for population growth at rate ν and productivity growth at a rate γ .

First, β . At the non-stochastic steady state, we have $R = \frac{1}{\beta}$. The average real interest rate in the U.S. is usually around 4% annually, which is about

1% quarterly. So $R - 1 = \frac{1}{\beta} - 1 = 0.01$, or $\beta \approx 0.99$. Cooley and Prescott follow a slightly more complex process and calibrate β as 0.987.

Next, θ . If you remember, $1 - \theta$ will be labour's share of output, a quantity that can be estimated from the national income accounts. Cooley and Prescott set θ to 0.40.

χ is set to one. Estimates from micro studies of the typical worker's intertemporal elasticity of substitution are in this range, and $\chi = 1$ gives us a simple Cobb-Douglas form of the utility function

$$u(c, h) = (1 - \alpha) \ln c + \alpha \ln(1 - h) \quad (5.18)$$

Now, solving for the steady state relationship between consumption, labour supply, and output, we get the following:

$$\frac{\alpha}{1 - h} = \frac{1 - \alpha}{c} w \quad (5.19)$$

$$= \frac{1 - \alpha}{c} (1 - \theta) \frac{y}{h} \quad (5.20)$$

α is set so that, in the non-stochastic steady state, 31% of available time is spent working ($h = 0.31$) and the steady state output to consumption ratio is about 1.33 ($c/y = 1.33$). Substituting into the equations above and solving for α , we get $\alpha = 0.64$.

Cooley and Prescott estimate that depreciation is 4.8% annually, so 1.2% quarterly ($\delta = 0.012$).

The parameters of the stochastic process for technology is easy to estimate. This model has perfect competition and constant returns to scale. So $z_t - z_{t-1}$ is the Solow residual. The average value of the Solow residual gives us our estimate for γ . Cooley and Prescott set $\gamma = 0.0156$, giving about 1.6% annual TFP growth. Once we subtract out this average, we can estimate an AR(1) model and get $\rho = 0.95$ and $\sigma_\epsilon = 0.007$.

Finally the quarterly population growth rate ν is set at 0.012.

Numerical solution of the model

Once we have set up the model, and calibrated parameters, we next need to find a numerical solution to the model.

- Demonstrate that the equilibrium of the model correspond to some planner's problem, that the solution to the planner's problem corresponds to some Bellman's equation, and apply numerical dynamic programming methods.
- Transform the problem itself using a linear-quadratic approximation around the steady state. Basically, you take a quadratic approximation of the objective function and a linear approximation of the constraints. The result will be a model with linear Euler equations.
- Log-linearization of the Euler equations around the steady state. This has the advantage that it leads to closed-form approximate results, so we can in the words of one researcher "inspect the mechanism." As with all linear approximations, this is a good approximation near the steady state but may be a poor approximation to how the system responds to large shocks.

Simulation

Under those assumptions we can simulate the model on a computer and we get time series for output, employment, productivity, investment, consumption, and capital. We can look at the correlation between any of these variables, the relative variance of different variables, etc.

There are many ways to do this simulation. The easiest is to take the Euler equations, take logs of everything, and linearize them around the balanced growth path. So the resulting variables are "detrended" - defined in terms of percentage deviation from trend/BGP. Once you have these simulated variables, you compare their properties to the properties of the corresponding detrended variables from the economy in question.

	US data		Baseline RBC model	
	Std. Dev (%)	$\rho_{x,GNP}$	Std. Dev (%)	$\rho_{x,GNP}$
GNP	1.72	1.0	1.35	1.0
Consumption (NDS)	0.86	0.77	0.329	0.843
Investment (GPDI)	8.24	0.91	5.954	0.992
Hours	1.59	0.86	0.769	0.986
Productivity	-	-	0.606	0.978

As you can see, the variability in output is a little smaller, but the same order of magnitude. As in reality, consumption is much less variable than output and investment is much more variable. Just like in real life, the model's variables are closely correlated with output. They're actually a little too closely correlated, which Cooley and Prescott note is due to the fact that there's only one source of uncertainty in this model.

To an RBC theorist, these numbers represent success. We've managed to write down a very simple model that duplicates many of the properties (moments) of the actual data. There are a few failures though. This model seems to understate the variability of both consumption and hours. The RBC approach to this failing is to investigate why the model doesn't match, and adjust the model so that it does match.

The consumption variability is simple. Even with careful measurement, a lot of "consumption" is actually purchase of consumer durables, which really belongs in investment.

5.1.5 Other issues

Variation in labor

For an example of how the dynamic GE approach is supposed to work, let's consider the low variation in hours. Gary Hansen, in a 1985 paper, developed an explanation and a fix. All of the variation in hours worked in this model take the form of changes in the hours worked by each worker. We know that most of the variation in hours over the business cycle take the form of variations in employment. Recall that the functional form of utility is:

$$u(c, h) = \frac{\left(c_t^{1-\alpha}(1-h_t)^\alpha\right)^{1-\chi} - 1}{1-\chi} \quad (5.21)$$

In order to generate higher variation in hours worked for each individual worker, we need to make them more willing to substitute intertemporally - work less when wages are low and more when they are high. For example, if $\chi = 0$, then their IES is infinity - they will work all day or not at all, depending on wages. But micro studies show a low IES, so we can't justify simply lowering χ .

Hansen's solution is to note that much of the variation in hours takes the form of variation in employment itself. He argues that it is not profitable to vary hours, given the fixed costs of employment. Instead he sets up the following model of the labor market. At the beginning of each period, a worker signs a binding contract to work h^* (say 8) hours with probability π_t and zero hours with probability $(1-\pi_t)$. Whether the worker works or not is decided by lottery. The worker gets paid wage $w_t\pi_t$ whether or not he works. He doesn't get to choose the value of h^* , but does choose π_t .

Expected utility is:

$$E[1 - \alpha \ln c + \alpha \ln(1 - h)] = (1 - \alpha) \ln c + \pi\alpha \ln(1 - h^*) \quad (5.22)$$

Now notice that this is linear in π_t . Per capita hours in this economy are $\pi_t h^*$, so the aggregate IES is quite high. It turns out that once you make this change, hours are much more variable, and the problem is "fixed".

Internal propagation mechanism

One of the most difficult problems for RBC adherents to deal comes from the results of Cogley and Nason (AER 1995). They analyze the baseline RBC model from a different perspective. One of the selling points of the RBC model is that fluctuations in the model are persistent. However, their persistence really isn't much more than that of the Solow residual, which is the exogenous source of shocks. Remember that the source of persistence in the baseline RBC model is that investment is higher in booms, so capital is higher in the near future. The problem is that new investment is very small relative to the capital stock, so the capital stock itself varies little.

Many papers since Cogley and Nason aim to find a better internal persistence mechanism. These include:

- Financial markets - for example suppose that an unexpected negative shock causes solvent but illiquid companies to become cashflow-constrained or even go bankrupt. They could then become less efficient as a result.
- Labor market search - suppose that it takes time for workers to find new jobs that they match well with.

Notice that these solutions tend to move the models away from the world of perfectly functioning markets and no motivation for intervention.

5.1.6 Criticisms

The RBC school of thought has grown substantially since the early papers. Part of the reason is that it's compelling as science. We take the model seriously, expect it to actually match observations quantitatively, and when it doesn't, we adjust the model. Another reason it's done so well is that the methodology is a natural article generator. Find something that hasn't been modeled yet, add it to the baseline model, and see what you get. The schools that focus in this area have turned out vast quantities of grad students over the last few years.

The main criticisms of the dynamic GE methodology are:

- Why would you consider matching the moments in the data a desirable property in a model that you know leaves out important features?
- Statistical tests are meaningless except in the context of an alternative. It may be the case that there are hundreds of different models that do equally well.

Some key criticisms of the specific hypothesis that fluctuations in technical progress explain business cycles are:

- If we are to identify the Solow residual as the measure of short-term technological progress, we must be prepared to interpret many deep recessions as exhibiting technical regress (especially the Great Depression or the recent troubles in Japan).
- Furthermore it is not clear what particular technological advances or disasters can be associated with any of the major short-term swings in the Solow residual.
- If the Solow residual measures what RBC model says it does, then it should be uncorrelated with the political party of the President, military purchases, and oil price movements (once energy usage has been accounted for). But it is correlated with all those things (Hall)

One approach that some proponents of the RBC theory have suggested is that the Solow residual is poorly measured. Capital (and labor) utilization rates tend to vary significantly and procyclically. If you use the capital stock to measure the flow of capital services, you will overstate the extent of fluctuations in technical progress. There's a paper by King and Rebelo in the Handbook of Macroeconomics that adds variable utilization to an RBC model and gets plausible fluctuations without negative Solow residuals.