PHYS 101 Final Examination - A

Saturday, 14 April, 2008 Name ____________________

Time: 3 hours Student number ____________________

No aids such as calculators, formula sheets are permitted.

Explain your reasoning with words and diagrams for problems 5-8.

1. For each of the following five questions, circle only one answer. (10 marks)

(i) An object moves with constant speed in a straight line. Which of the following statements must be true?

- (a) No forces act on the object.
- (b) A single constant force acts on the object in the direction of motion.
- (c) The net force acting on the object depends on the value of the speed.
- (d) The net force acting on the object is zero.
- (e) The net force acting on the object cannot be determined.

(ii) A falling object experiences a drag force due to air resistance. Which statement is NOT true?

- (a) The drag force depends on the falling speed.
- (b) The faster the object falls, the stronger the air resistance.
- (c) The mechanical energy of the object is conserved.
- (d) The speed of the object will reach a maximum value and then stop changing.
- (e) The net force acting on the object will eventually reach zero.

(iii) In the absence of friction, if an object is subject to a constant net force F, which of the following statements is true? The distance it moves when starting from rest:

- (a) increases with the square of the elapsed time
- (b) is proportional to its mass
- (c) is independent of F
- (d) increases linearly with time
- (e) is proportional to F^2.

(iv) Subject to a constant force from its brakes, a car moving at an initial speed v_i slows uniformly to a stop while traveling on a level surface. What fraction of its initial kinetic energy remains when its speed is $v_i/2$?

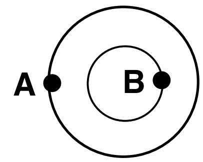
(a) 3/4 (b) ½ (c) ¼ (d) 1/8 (e) none of [a-d]

(v) A car of mass m travels at a speed v towards a stationary truck of mass $3m$. What is the speed of the centre-of-mass of the system?

(a) v (b) $v/2$ (c) $3v/4$ (d) $v/3$ (e) $v/4$

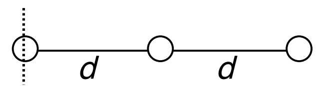
2. For each of the following five questions, circle only one answer. (10 marks)

(i) Two objects A and B rotate around a common centre with the same angular frequency. Their masses are the same, but the radius of A's orbit is twice that of B. What is the ratio L_A/L_B, where L is the angular momentum?



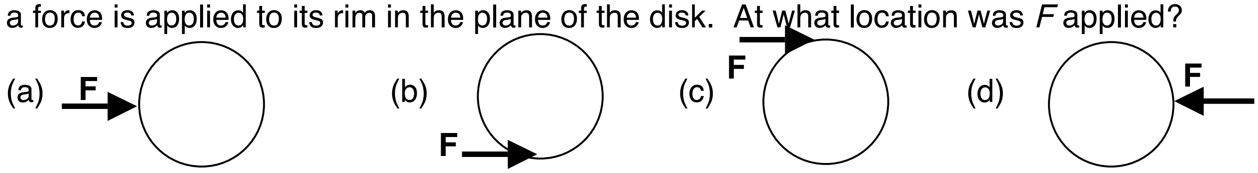
(a) 4 (b) 2 (c) 1 (d) ½ (e) ¼

(ii) Three point objects, each of mass m, lie in a line as shown, connected by massless rigid rods of length d. What is the magnitude of their moment of inertia about an axis which runs through one of the end masses and is perpendicular to their axis of symmetry?



(a) md^2 (b) $2md^2$ (c) $3md^2$ (d) $4md^2$ (e) $5md^2$

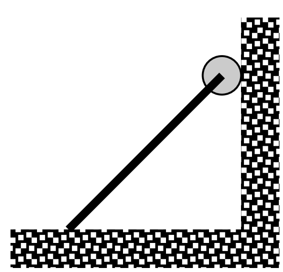
(iii) A disk of mass M, radius R, experiences an angular acceleration $\alpha = +2F/MR$ when a force is applied to its rim in the plane of the disk. At what location was F applied?



(iv) A point P is at a distance R from the axis of rotation of a rigid body whose angular velocity and angular acceleration are ω and α respectively. The linear speed, centripetal acceleration and tangential acceleration of the point can be expressed as:

	Linear speed	Centripetal acceleration	Tangential acceleration
(a)	$R\omega$	$R\omega^2$	$R\alpha$
(b)	$R\omega$	$R\alpha$	$R\omega^2$
(c)	$R\omega^2$	$R\alpha$	$R\omega$
(d)	$R\omega$	$R\omega^2$	$R\omega$
(e)	$R\omega^2$	$R\alpha$	$R\omega^2$

(v) The upper end of a rod of mass m is attached to a wall by a frictionless hinge, as shown, while its lower end rests on a frictionless surface. What is the magnitude of the force on the lower end of the rod?



(a) mg (b) larger than mg (c) 0 (d) less than mg (e) none of (a-d)

3. For each of the following five questions, circle only one answer. (10 marks)

(i) To double the period of a simple pendulum, the length
- (a) must be increased by a factor of 2.
- (b) must be decreased by a factor of 2.
- (c) must be increased by a factor of $\sqrt{2}$.
- (d) must be increased by a factor of 4.
- (e) is irrelevant: pendulum length is not related to period.

(ii) A string fixed at both ends is vibrating in a standing wave. The distance between two consecutive nodes is d. The wavelength is

(a) $d/2$ (b) d (c) $3d/2$ (d) $2d$ (e) $4d$

(iii) A longitudinal wave is distinguished from a transverse wave by the fact that in longitudinal waves
- (a) the particle vibration is parallel to the direction of propagation.
- (b) the particle vibration is perpendicular to the direction of propagation.
- (c) energy is transported from one point in space to another point.
- (d) vibrations occur only in air or water.
- (e) energy is not transported from one point in space to another point.

(iv) Two waves are traveling through the same medium, one with frequency 200 s^{-1} and one with frequency 300 s^{-1}. What is their beat frequency?

(a) 500 s^{-1} (b) 200 s^{-1} (c) 100 s^{-1} (d) 300 s^{-1} (e) 0

(v) An object undergoes simple harmonic motion in the x-direction starting with its maximum speed at $t = 0$ and $x = 0$. Initially moving to the right, it reaches its maximum displacement $x = 1$ m after 2 seconds. Its motion is described by

(a) $\sin(\pi t)$ (b) $\sin(\pi t/4)$ (c) $\sin(t/2)$ (d) $\sin(t/8)$ (e) none of [a-d]

4. For each of the following five questions, circle only one answer. (10 marks)

(i) Blood flows from a large artery of radius 0.3 cm, where its speed is 8 cm/s, into a region where the radius has been reduced to 0.2 cm because of thickening of the walls. Assuming the blood to be incompressible, what is its speed in the narrower region in cm/second? Assume the viscosity is very small.

(a) 18 (b) 0.06 (c) 12 (d) 5.3 (e) 3.6

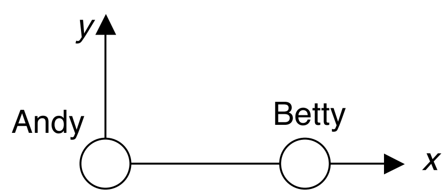
(ii) In buoyancy, what does it mean when an object has a negative effective weight?
- (a) the object will continue to sink
- (b) the object is near a black hole
- (c) the object has a variable shape
- (d) the density of the fluid is less than the density of the object
- (e) the buoyant force exceeds the object's weight.

(iii) When inspecting the heating system of an old building, an engineer notices that many small pipes flow in parallel towards the furnace. She proposes that the small pipes be replaced by a larger pipe of twice the diameter. How many small pipes can the large one replace, and still maintain the same flow of fluid? Assume that the fluid is viscous, and that the pressure drop per unit length of pipe is the same in both cases.
(a) 16 (b) 8 (c) 4 (d) 32 (e) 2

(iv) A vase of fresh flowers is placed on a table. Judy, who is sitting 1 m from the flowers, smells their scent in 2 seconds. How long, in seconds, does it take the scent to arrive at Jasmine, who is sitting 3 m away?
(a) 18 (b) 9 (c) 3 (d) 2 (e) 27

(v) A spherical object with mass m and radius R is observed to have a diffusion constant D in a particular fluid. If the mass is doubled, but the size of the object remains unchanged, what diffusion constant would it obey?
(a) $4D$ (b) $D/2$ (c) $D/4$ (d) $2D$ (e) D

5. Andy and Betty walk in straight lines at constant speed $v_A = v_B = 1$ m/s, although their velocities $\mathbf{v}_A$ and $\mathbf{v}_B$ are different. Andy walks straight east (along the x-axis) starting at $x_A = y_A = 0$ at $t = 0$. Betty walks north (parallel to the y-axis) starting at $x_B = 10$ m, $y_B = 0$ at $t = 0$.



(a) What are the general expressions for their motion [$x_A(t)$, $y_A(t)$, $x_B(t)$, $y_B(t)$]?
(b) What is the square of the separation s between them?
(c) Find a time t when $s^2 = 0$, or show mathematically why it can never be so.
(d) Explain in words what your answer in part (c) means. (15 marks)

6. A disk of uniform density and moment of inertia I_1 spins about its axis at an angular frequency ω_o. A second disk of the same mass and radius is dropped on the first one, with the result that the two of them rotate together at the same angular frequency $0.4\ \omega_o$.
(a) What is the moment of inertia of the second disk, in terms of I_1?
(b) What fraction of the kinetic energy, if any, is lost once the disks stick together?
(c) Is the mass distribution in disk #2 uniform? If not, is it more dense at the rim or in the centre, compared to a uniform distribution? (15 marks)

7. An airplane has a mass of 2×10^6 kg and its wings have a combined area of 400 m^2 on their lower surface.
(a) What pressure difference is required between the upper and lower surfaces of the wings to permit the airplane to fly? Use $g = 10$ m/s^2.
(b) If the air speed across the bottom surface is 200 m/s, what must be the air speed over the top to permit the plane to fly near sea level with $\rho_{air} = 1$ kg/m^3.

(c) Repeat your calculation in (b) for a density ρ_{air} = 0.2 kg/m^3 expected at very high elevations. Do you think that a wing shape to generate such a speed difference is achievable? (14 marks)

8. (a) A particular insect flies at a constant speed of 1 m/s, and changes direction every 3 seconds. How long would it take for the end-to-end displacement of its random motion to equal 10 m?
(b) Suppose that the insect emitted a scent that was detectable by another of its species even at very low concentrations. The molecule of the scent travels at 300 m/s, but changes direction through collisions every 10^{-11} s. Once the molecule has been released by the insect, how long would it take for the end-to-end displacement of its trajectory to equal 10 m?
(c) Would a second insect 10 m away know of the presence of the first one before it had traveled 10 m in displacement? (There could be pros and cons to this result, depending on whether the insects are male or female).
(d) Suppose that a small rodent could run on the ground at the same speed as the insect flew: the rodent is confined to move in two dimensions whereas the insect can fly in three dimensions. Is the time taken for the rodent's trajectory to have the same end-to-end displacement greater than, equal to, or less than, the flight time of the insect? (16 marks)

Answers:
1. *d, c, a, c, e*.
2. *a, e, b, a, d*.
3. *d, d, a, c, b*.
4. *a, e, a, a, e*.
5. (a) $x_A(t) = t$ $y_A(t) = 0$; $x_B(t) = 10$ $y_B(t) = t$ (b) $s^2 = 2t^2 - 20t + 100$
(c) $t(\text{at } s=0) = \{10 \pm (-100)^{1/2}\} / 2$.
6. (a) $I_2 = 3/2\ I_1$ (b) $K_{final} = 2/5\ K_{initial}$ (c) *disk 2 has more mass at the rim.*
7. (a) $\Delta P = 5x10^4$ N (b) $v_{upper} = 374$ m/s (c) $v_{upper} = 734$ m/s.
8. (a) *33 s* (b) $1.1x10^8$ s (c) *unaware of scent* (d) $<r_{ee}^2>$ *is independent of dimension*

Fundamental formulae

Linear kinematics and dynamics

$\Delta v = at$ $\Delta x = v_o t + at^2/2$ $\Delta x = (v_f^2 - v_i^2)/2a$ $a_c = v^2/R$ $\mathbf{F} = m\mathbf{a}$ $R = (v_o^2/g)\sin(2\theta)$

Forces $\mathbf{F} = -k\mathbf{x}$ $F = Gm_1m_2/r^2$ $f = \mu N$ $F = (C_D/2)\rho A v^2$

$F = 6\pi\eta R v$ $v(t) = v_o \exp(-c_1 t/m)$ $\Delta x = (mv_o/c_1)\,[1 - \exp(-c_1 t/m)]$

Work, energy and power

$W = \mathbf{F}\cdot\Delta\mathbf{x}$ $W_{\text{non-con}} = \Delta(K + U)$ $F = -dU/dx$ $P = \mathbf{F}\cdot\mathbf{v}$

$U(r) = -Gm_1m_2/r$ $U(x) = kx^2/2$

Rigid objects

$\mathbf{r}_{cm} = \Sigma_i m_i \mathbf{r}_i / M_{tot}$ $\mathbf{v}_{cm} = d\mathbf{r}_{cm}/dt$ $\mathbf{a}_{cm} = d\mathbf{v}_{cm}/dt$ $\mathbf{F}_{tot} = M_{tot}\mathbf{a}_{cm}$

Rotational kinematics and dynamics

$\Delta\omega = \alpha t$ $\Delta\theta = \omega_o t + \alpha t^2/2$ $\Delta\theta = (\omega_f^2 - \omega_i^2)/2\alpha$ $L = r_\perp p = I\omega$

$v = \omega R$ $a_{tan} = \alpha R$ $\tau = r_\perp F$ $\tau = dL/dt = I\alpha$

$K = I\omega^2/2$ $I = \Sigma_i m_i r_{\perp i}^2$ $I_{ring} = MR^2$ $I_{disk} = MR^2/2$

Oscillations

$y(t) = A\sin(\omega t)$ $v(t) = \omega A\cos(\omega t)$ $a(t) = -\omega^2 A\sin(\omega t)$ $a = -\omega^2 x$

$v = (F/\mu)^{1/2}$ $\lambda_n = 2L/n$ $y(x,t) = A\sin(2\pi x/\lambda \; -/+ \; 2\pi t/T)$

$\omega = 2\pi f = 2\pi/T$ $\omega = (k/m)^{1/2}$ $\omega = (g/L)^{1/2}$ $speed = f\lambda$

$Power = \mu\omega^2 A^2 v/2$

Fluids

$Pressure = P = force/area$ $P = P_o + \rho g h$ (static) $B = \rho_L g V$

$\rho v A = constant$ $P + \rho v^2/2 + \rho g h = [constant]$ (dynamic)

$Volume\ flow = Q = v_{av}A$ $Q = (\pi R^4/8\eta)\cdot(\Delta P/L)$ (streamline, drag)

Random walks

$\langle r_{ee}^2\rangle = Nb^2$ (random) $\langle r_{ee}^2\rangle = 6Dt$ (3D) $D = k_B T/(6\pi\eta R)$ (Stokes-Einstein)

Thermal expansion

$\Delta L/L = \alpha\,\Delta T$ $\Delta V/V = \beta\,\Delta T$ $\beta = 3\alpha$

derivative of a polynomial: $dx^n/dt = n\,x^{n-1}$