Lecture 11 - Conservative forces

What's important:

- conservation of energy; power

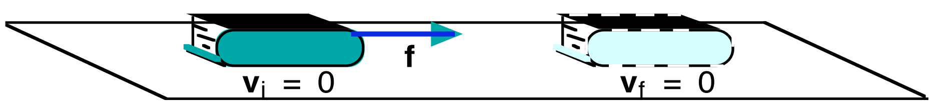
Demonstrations: none

Conservative Forces

The concept of potential energy that we introduced in the last lecture has an aspect of reversibility associated with it:

- we do work to slide a block up an inclined plane (increase U)
- gravity does work as a block slides down an inclined plane (decrease U).

What about a force like friction? We can slide a book across a table against a frictional force



We do work on the book, $W \neq 0$, but $v_i = v_f = 0$ and there is no change in K: $\Delta K = 0$.

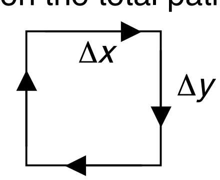
Further, there is no change in U: after we have stopped pushing the book, it does not move back into its original position (*i.e.* the potential energy of the book hasn't changed, so the book can't reduce its potential energy by moving to its original position)! So, friction does not have a potential energy U associated with it.

We say that gravity is a **conservative force**: it has a potential energy which depends on position. Friction is a **non-conservative (or dissipative) force** with no potential energy.

Are there other differences between conservative and non-conservative forces?

Gravity
$W = U_f - U_i$
depends only
on the end-points

Friction
$W = f\,\Delta x + f\,\Delta y + f\,\Delta x + f\,\Delta y$
depends on the total path taken



So, in a conservative force, the work depends only on end-points of the path; in a non-conservative force, the work depends on the path.

Finally, we can generalize the conservation of energy relation to read:

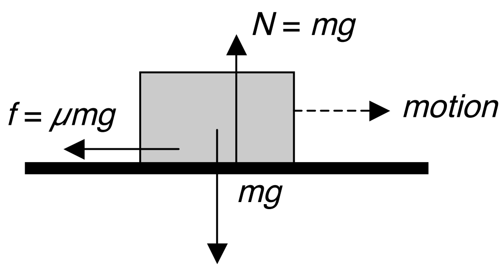
$$W_{\text{non-cons}} = \Delta E = \Delta(K + U)$$

↑

all the conservative work
disappears into potential energy

Example A block of mass m has an initial speed v_i. It slides on a table, subject to friction μ_k. How far does the block move before stopping? Do two calculations: forces and energy.

<u>force approach</u>



frictional force on the object is constant, producing an acceleration

$ma = -\mu mg$ or $a = -\mu g$.

From kinematics for objects with constant acceleration,

$x = (v_f^2 - v_i^2) / 2a$ → $x = -v_i^2 / 2a$

becomes

$x = -v_i^2 / 2(-\mu g) = v_i^2 / 2\mu g$

<u>energy approach</u>

change in kinetic energy = D$K = mv_f^2/2 - mv_i^2/2 = -mv_i^2/2$.

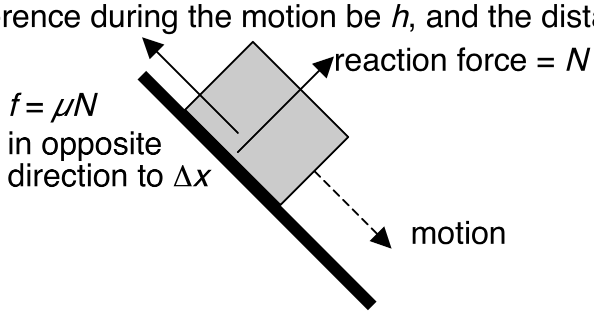
non-conservative work = $-f \cdot x = -\mu mg\, x$.

then, from $W_{\text{non-con}} = \Delta(K + U) = \Delta K$

$-\mu mg\, x = -mv_i^2/2$ or $x = v_i^2 / 2\mu g$

Note that the heat from friction is absorbed by both the block and the table; it has already been taken into account in both calculations.

Example block starting from rest and sliding down a plane with or without friction. Let the height difference during the motion be h, and the distance along the plane L.



change in kinetic energy = $\Delta K = mv_f^2 / 2$ positive
change in potential energy = $\Delta U = -mgh$ negative

no friction

$W_{\text{non-con}} = 0$ $\Delta(K + U) = 0$
$\Delta K = -\Delta U$
$mv_f^2 / 2 = -(-mgh)$ or $v_f^2 / 2 = gh$.

with friction

$W_{\text{non-con}} = -\mu NL$ $\Delta(K + U) = -\mu NL$
$\Delta K = -\Delta U - \mu NL$
$mv_f^2 / 2 = -(-mgh) - \mu NL$ or $v_f^2 / 2 = gh - \mu NL/m$.

In other words, v_f is less with friction than without: the object slides more slowly in the presence of friction, as we expect.

In both of these examples, friction has done *negative* work to *lower* the total mechanical energy **E** of the system.

Potential and Force

We said that the work done by the system lowers its potential

$$W_{\text{by the system}} = -\Delta U$$

Suppose now we let Δx become sufficiently small that the force is constant over the x range. Then

$$W = F\,\Delta x$$

whence

$$F\,\Delta x = -\Delta U$$

or

$$F = -\Delta U / \Delta x \qquad \text{as } \Delta x \to 0$$

That is, force is the (negative) rate of change of potential energy with distance.

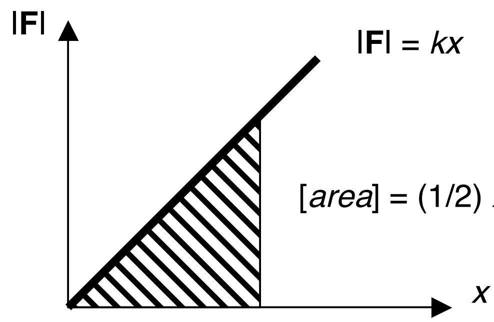
In three dimensions, this equation applies component by component:

$$F_x = -\Delta U / \Delta x \qquad F_y = -\Delta U / \Delta y \qquad F_z = -\Delta U / \Delta z$$

Clearly, this only applies for conservative forces.

Example:

Springs provide a simple example of a force which depends on distance: |**F**| = kx. We find the work required to stretch a spring by taking the area under the force-distance curve:



As usual, the area of the triangle under the curve is just

[*area*] = 1/2 [*base*] • [*height*] = 1/2 $x \cdot kx = kx^2/2$.

In other words, the work required to stretch the spring is $kx^2/2$, and this increases the potential energy of the spring by

$U = kx^2/2$.

We can work from U to F by means of $F = -\Delta U / \Delta x$ (need the derivative of a polynomial to do this).

Power

Power is the rate of change of energy

$P = \Delta E / \Delta t$ as $\Delta t \to 0$.

In MKSA, power has units of Joules / seconds ≡ watts (1 hp = 1 horsepower = 746 watts). Your electric power utility often quotes energy in terms of *power x time*:

1 kW - hr = 1000 • 3600 watt-seconds

= 3.6×10^6 J.

If we are considering the power delivered by a system doing work, then

$P = W / \Delta t = (F \ \Delta x) / \Delta t = F (\Delta x / \Delta t)$

But

$\Delta x / \Delta t = v$,

so

$P = Fv$.

In three dimensions:

$P = \mathbf{F} \cdot \mathbf{v}$.

Example: The food intake in a typical diet releases 2500 Calories per day. What is the power generated in watts?

Energy units: 1 Calorie = 1000 calories = 1 kcal (note C *vs*. c).
1 calorie = 4.186 J

Hence: 2500 Calories = 2.5×10^6 calories = $2.5 \times 10^6 \times 4.186$ J

or

energy produced = 1.05×10^7 J

This energy is produced in 1 day = 24 x 60 x 60 = 8.64×10^4 seconds. Hence,

power = energy / time = $1.05 \times 10^7 / 8.64 \times 10^4$ = 121 J/s

The energy released is somewhat more than a 100 watt light bulb.

Example: Calculate the quadratic drag force experienced by a car, and the power required to overcome this force, for the following conditions:

(a) 90 km/hr

(b) 120 km/hr.

Take the car to have a cross sectional area of 2 m^2, and a drag coefficient of 0.4. Quote your answer in hp.

Ans. (a) 10.8 hp, (b) 25.6 hp.