Lecture 11 - Conservative forces

What’s important:
• conservation of energy; power

Demonstrations: none

Conservative Forces

The concept of potential energy that we introduced in the last lecture has an aspect of reversibility associated with it:
• we do work to slide a block up an inclined plane (increase $U$)
• gravity does work as a block slides down an inclined plane (decrease $U$).

What about a force like friction? We can slide a book across a table against a frictional force

We do work on the book, $W \neq 0$, but $v_i = v_f = 0$ and there is no change in $K$: $\Delta K = 0$.

Further, there is no change in $U$: after we have stopped pushing the book, it does not move back into its original position (i.e. the potential energy of the book hasn’t changed, so the book can’t reduce its potential energy by moving to its original position)! So, friction does not have a potential energy $U$ associated with it.

We say that gravity is a conservative force: it has a potential energy which depends on position. Friction is a non-conservative (or dissipative) force with no potential energy.

Are there other differences between conservative and non-conservative forces?

<table>
<thead>
<tr>
<th>Gravity</th>
<th>Friction</th>
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<tbody>
<tr>
<td>$W = U_f - U$</td>
<td>$W = f \Delta x + f \Delta y + f \Delta x + f \Delta y$</td>
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<tr>
<td>depends only on the end-points</td>
<td>depends on the total path taken</td>
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So, in a conservative force, the work depends only on end-points of the path; in a non-conservative force, the work depends on the path.
Finally, we can generalize the conservation of energy relation to read:

\[ W_{\text{non-cons}} = \Delta E = \Delta (K + U) \]

all the conservative work disappears into potential energy

**Example**  A block of mass \( m \) has an initial speed \( v_i \). It slides on a table, subject to friction \( \mu_k \). How far does the block move before stopping? Do two calculations: forces and energy.

**force approach**

frictional force on the object is constant, producing an acceleration

\[ ma = -\mu mg \quad \text{or} \quad a = -\mu g. \]

From kinematics for objects with constant acceleration,

\[ x = \frac{(v_f^2 - v_i^2)}{2a} \quad \rightarrow \quad x = -\frac{v_i^2}{2a} \]

becomes

\[ x = -\frac{v_i^2}{2(-\mu g)} = \frac{v_i^2}{2\mu g} \]

**energy approach**

change in kinetic energy = \( \Delta K = \frac{mv_f^2}{2} - \frac{mv_i^2}{2} = -\frac{mv_i^2}{2} \).

non-conservative work = \( -f \cdot x = -\mu mg \cdot x \).

then, from \( W_{\text{non-cons}} = \Delta (K + U) = \Delta K \)

\[ -\mu mg \cdot x = -\frac{mv_i^2}{2} \quad \text{or} \quad x = \frac{v_i^2}{2\mu g} \]

Note that the heat from friction is absorbed by both the block and the table; it has already been taken into account in both calculations.

**Example**  block starting from rest and sliding down a plane with or without friction. Let the height difference during the motion be \( h \), and the distance along the plane \( L \).
change in kinetic energy = $\Delta K = \frac{mv_f^2}{2}$ positive
change in potential energy = $\Delta U = -mgh$ negative

no friction
$W_{\text{non-con}} = 0$
$\Delta(K + U) = 0$
$\Delta K = -\Delta U$
$mv_i^2 / 2 = -(mgh)$ or $v_i^2 / 2 = gh.

with friction
$W_{\text{non-con}} = -\mu NL$
$\Delta(K + U) = -\mu NL$
$\Delta K = -\Delta U - \mu NL$
$mv_i^2 / 2 = -(mgh) - \mu NL$ or $v_i^2 / 2 = gh - \mu NL/m.$

In other words, $v_i$ is less with friction than without: the object slides more slowly in the presence of friction, as we expect.

In both of these examples, friction has done negative work to lower the total mechanical energy $E$ of the system.

**Potential and Force**

We said that the work done by the system lowers its potential $W_{\text{by the system}} = -\Delta U$

Suppose now we let $\Delta x$ become sufficiently small that the force is constant over the $x$ range. Then
$W = F \Delta x$

whence
$F \Delta x = -\Delta U$

or
$F = -\Delta U / \Delta x$ as $\Delta x \to 0$

That is, force is the (negative) rate of change of potential energy with distance.

In three dimensions, this equation applies component by component:

$F_x = -\Delta U / \Delta x$  
$F_y = -\Delta U / \Delta y$  
$F_z = -\Delta U / \Delta z$

Clearly, this only applies for conservative forces.
Example:

Springs provide a simple example of a force which depends on distance: $|\mathbf{F}| = kx$. We find the work required to stretch a spring by taking the area under the force-distance curve:

\[
\text{As usual, the area of the triangle under the curve is just}
\]
\[
\text{[area]} = \frac{1}{2} \text{[base]} \cdot \text{[height]} = \frac{1}{2} x \cdot kx = \frac{kx^2}{2}.
\]

In other words, the work required to stretch the spring is $\frac{kx^2}{2}$, and this increases the potential energy of the spring by $U = \frac{kx^2}{2}$.

We can work from $U$ to $F$ by means of $F = -\Delta U / \Delta x$ (need the derivative of a polynomial to do this).

Power

\textit{Power is the rate of change of energy}

\[ P = \frac{\Delta E}{\Delta t} \quad \text{as } \Delta t \to 0. \]

In MKSA, power has units of Joules / seconds = watts (1 hp = 1 horsepower = 746 watts). Your electric power utility often quotes energy in terms of power x time:

\[ 1 \text{ kW - hr} = 1000 \cdot 3600 \text{ watt-seconds} = 3.6 \times 10^6 \text{ J}. \]

If we are considering the power delivered by a system doing work, then

\[ P = W / \Delta t = (F \Delta x) / \Delta t = F (\Delta x / \Delta t) \]

But

\[ \Delta x / \Delta t = v, \]

so

\[ P = Fv. \]
In three dimensions:
\[ P = F \cdot v. \]

*Example:* The food intake in a typical diet releases 2500 Calories per day. What is the power generated in watts?

Energy units: 

- 1 Calorie = 1000 calories = 1 kcal (note C vs. c).
- 1 calorie = 4.186 J

Hence: 2500 Calories = \(2.5 \times 10^6\) calories = \(2.5 \times 10^6\) x 4.186 J

or

energy produced = \(1.05 \times 10^7\) J

This energy is produced in 1 day = 24 x 60 x 60 = \(8.64 \times 10^4\) seconds. Hence,

\[
\text{power} = \frac{\text{energy}}{\text{time}} = \frac{1.05 \times 10^7}{8.64 \times 10^4} = 121 \text{ J/s}
\]

The energy released is somewhat more than a 100 watt light bulb.

*Example:* Calculate the quadratic drag force experienced by a car, and the power required to overcome this force, for the following conditions:

(a) 90 km/hr
(b) 120 km/hr.

Take the car to have a cross sectional area of 2 m\(^2\), and a drag coefficient of 0.4. Quote your answer in hp.

*Ans.* (a) 10.8 hp, (b) 25.6 hp.