Lecture 18/19 - Statics

What's important:
•conditions for no rotation and no translation
Demonstrations: mass hung from a bar, two scales at the end; T-bar with different locations for hanging mass; meter stick

Statics

In linear kinematics, we said that for an object to be stationary, its acceleration **a** must vanish. Applying the dynamics laws

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}} \quad \text{and} \quad \vec{\tau} = I\vec{\alpha}$$

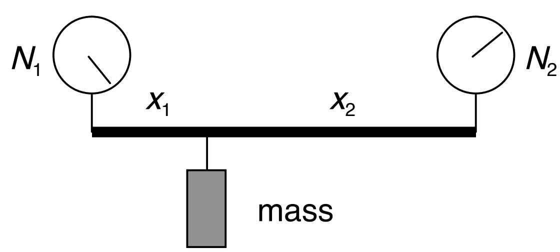
we find that a stationary object (or one in uniform motion) must satisfy

$$\sum_i \vec{\mathbf{F}}_i = 0$$ for static equilibrium (up to 3 independent equations).

$$\sum_i \vec{\tau}_i = 0$$ for static equilibrium (up to 3 equations)

Note that if the object is not rotating, then the net torque around *every* axis is zero. But this doesn't mean that there are an infinite number of torque equations, for an infinite number of axes - there are still only three conditions.

Example 1
A mass *m* is hung from a horizontal rod which is suspended at each end by scales. Slide the mass along the rod and note the different readings on the scales.



The scale readings tells us the normal or reaction force at each end of the bar. The condition of no translation in the *y*-direction yields

$$mg = N_1 + N_2 \tag{1}$$

But because there are 2 unknowns in the problem, N_1 and N_2, Eq. (1) does not contain enough information to solve the problem. The condition that there be no rotation, meaning no net torque, provides a second constraint which allows the problem to be solved.

Clockwise torque = (-) x_1N_1

Counter clockwise torque = x_2N_2

Sum of torques equals zero:

$$-x_1N_1 + x_2N_2 = 0$$

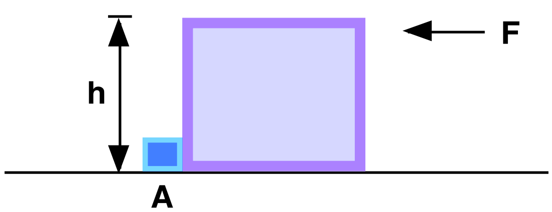
or

$$x_1N_1 = x_2N_2. \quad (2)$$

Demo Place a meter stick (or pencil) level on the index finger of each hand. Slowly move the hands towards each other until the fingers almost touch. The fingers will be close to the centre of the stick.

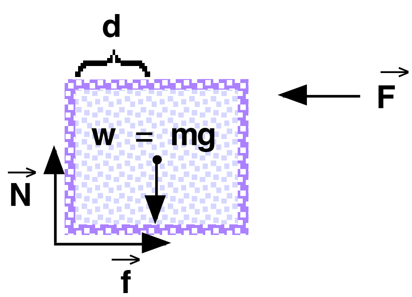
What's happening? The torque equation tells us the reaction forces will vary inversely with the distance from the stick's CM location. Thus, the reaction force furthest from the CM will be less, and its friction force (from the finger) will also be less. Preferentially, the finger furthest away will move towards the centre until it experiences the same torque as the motionless finger, and the same friction force. Actually, it will overshoot a little because of the difference between μ_k and μ_s. Once it has overshot, the moving finger will stop and the stationary one will begin to slide. It may be easiest to see the effect by holding one finger completely stationary, so one can see how the bar itself slides.

Example 2 A block is pushed against a fixed point **A:**



Find the maximum force before the block rotates.

First, construct a free-body diagram. In addition to the applied force **F**, there is a normal force **N** opposing the weight m**g**, and there is a reaction force **f** opposing the applied force **F**:



Note, $\vec{\mathbf{N}}$ and $\vec{\mathbf{f}}$ act at point **A** when block is about to rotate over stopper.

We can use the linear conditions $\Sigma_i \mathbf{f}_i = 0$ to establish the constraints

x-direction $\mathbf{f} = -\mathbf{F}$

y-direction $\mathbf{N} = -m\mathbf{g} = -\mathbf{w}$

We impose the condition of no rotational motion by saying that there should be no net torque:

$$\Sigma_i \tau_i = 0$$

Clockwise (negative, τ into plane)

$$\tau_{CW} = -dmg$$

Counterclockwise (positive, τ out of plane)

$$\tau_{CCW} = +hF.$$

Adding, and equating to zero:

$$\tau_{CW} + \tau_{CCW} = 0$$

or

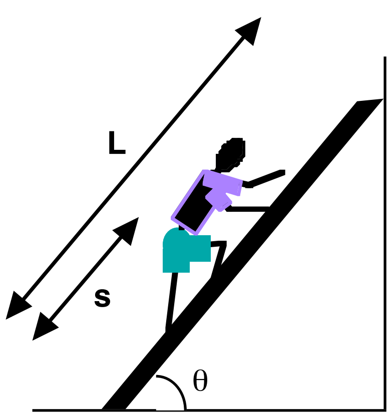
$$-dmg + hF = 0$$

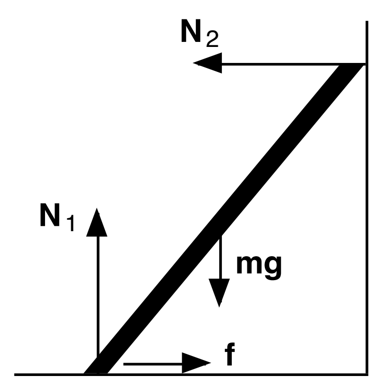
implies

$$F = (d/h)\ mg.$$

As expected, the larger the value of d/h, the larger the applied force before toppling - SUV effect.

Example 3 A man climbs a massless ladder leaning against a frictionless wall. If the coefficient of friction between the ladder and the floor is μ_s, how far up the ladder can the man climb before the ladder slips?





From the condition of no rotation:

clockwise = counter-clockwise (magnitudes only, opposite signs)

$$mg\ s\ \cos\theta = N_2\ L\ \sin\theta \quad (1)$$

We need to find N_2:

- from translation in the x-direction $N_2 = f$ (magnitude)
- from friction at the slipping point $f = \mu N_1$ (magnitude)
- from translation in the y-direction $N_1 = mg$ (magnitude)
- taking all of these three relations: $N_2 = f = \mu N_1 = \mu mg$. (2)

Placing (2) into (1) gives

$$mg\, s \cos\theta = \mu mg\, L \sin\theta$$

Solving:

$$s \cos\theta = \mu\, L \sin\theta$$

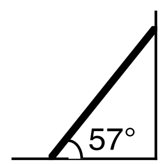
or

$$s / L = \mu \tan\theta.$$

Hence, for the person to climb the ladder all the way to **s = L**,

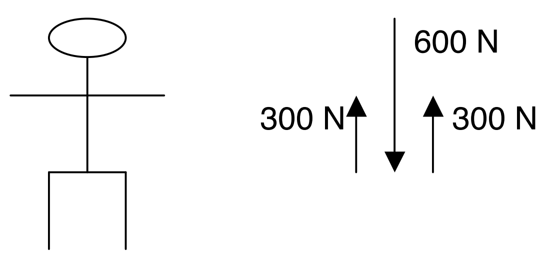
$$\tan\theta \geq \frac{1}{\mu}$$

If $\mu = 0.6 \Rightarrow \theta \simeq 1$ rad $\simeq 57°$



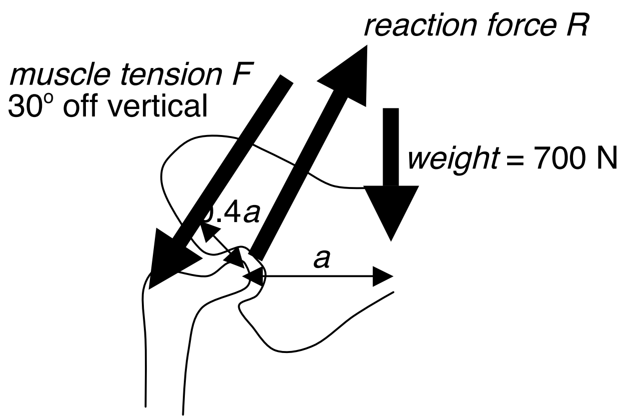
Example 4 Compare the forces on the top of the femur for a person of weight 800 N in two situations:
(a) relaxed stance on two legs
(b) standing on one leg only
(data are from Tuszynski and Dixon *Biomedical Applications of Introductory Physics*)

(a) relaxed stance
A person's leg accounts for about 1/8 of their mass, so the weight of each leg is roughly 100 N in this example, and the remainer of the body is 600 N. If this weight is symmetrically distributed over two legs, the reaction force from the femur is 300 N from each leg.



(b) on one leg

This is a balance of torques problem. Crudely, the dimensions around the pelvis are:



The body weight, acting through its centre of mass, is 700 N and is a perpendicular distance *a* from the top of the femur. Counteracting this weight is a tension *F* in a group of muscles running from the femur to the pelvis; we'll represent this as a single force oriented 30° with respect to the vertical, a distance 0.4*a* from the femur.

Balancing torques:

$0.4a \cdot F = 700\ a$ (in N•m)

or

$F = 700 / 0.4 = 1750$ N.

Balancing forces:

$R_x = F \sin 30^\circ = 1750 \times 0.5 = 875$ N

$R_y = F \cos 30^\circ + w = 1750 \times (\sqrt{3}/2) + 700 = 2216$ N.

Thus, the total reaction force is

$R = (875^2 + 2216^2)^{1/2}$

$= 2380$ N.

In other words, the reaction force on the femur is about 8 times as high when standing on one leg than when standing relaxed on two legs.