Lecture 2: Acceleration

What's important:
- relationships between displacement, velocity, acceleration
- permutations of two basic equations for motion in one dimension
- vectors

Demonstrations: none

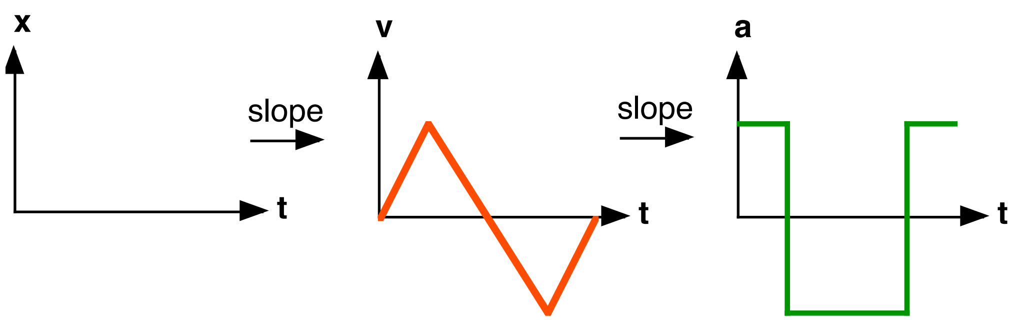
Acceleration

We can go to higher order rates of change by looking at the rate of change of the velocity (since $s = |\mathbf{v}|$, then we will deal with velocities rather than speeds).

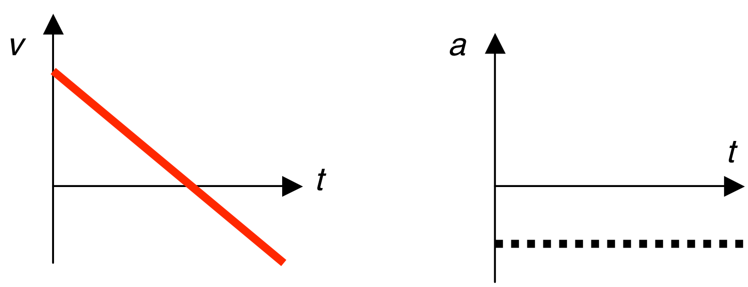
$$[\textit{average acceleration}] = \bar{\mathbf{a}} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{t_2 - t_1} \qquad \text{(independent of path)}$$

$$[\textit{instantaneous acceleration}] = \mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t} \qquad \text{as } \Delta t \rightarrow 0 \;\; \text{(which becomes } \mathbf{a} = d\mathbf{v}/dt\text{)}$$

In terms of graphs:



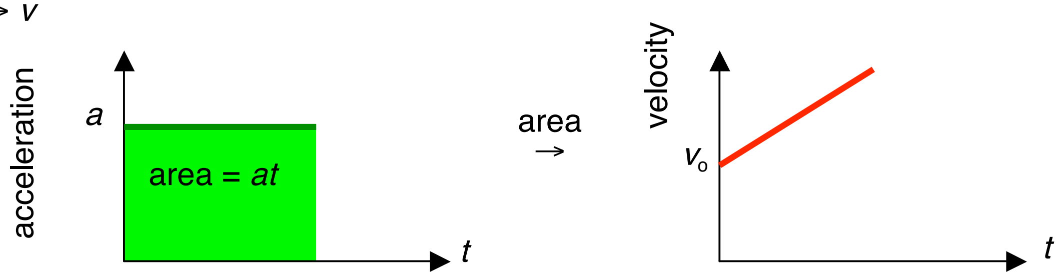
For example, suppose that the motor fails on a boat heading into the wind:



Constant acceleration in one dimension

To obtain x from v, or v from a (that is, to proceed in the opposite direction from the rates), one takes areas under the curves of time-evolution graphs. For example, a car travelling at a constant speed of 100 km/hr (s) covers a distance of 100 km (st) in a time of 1 hour (t). The distance is the product of s and t, and is the area under the s *vs* t graph. We apply this to constant acceleration in one dimension

$a \rightarrow v$



The area under the curve gives the change in v (that is, $\Delta v = v - v_o$), NOT v itself. From the graph of **constant acceleration** *vs* t,

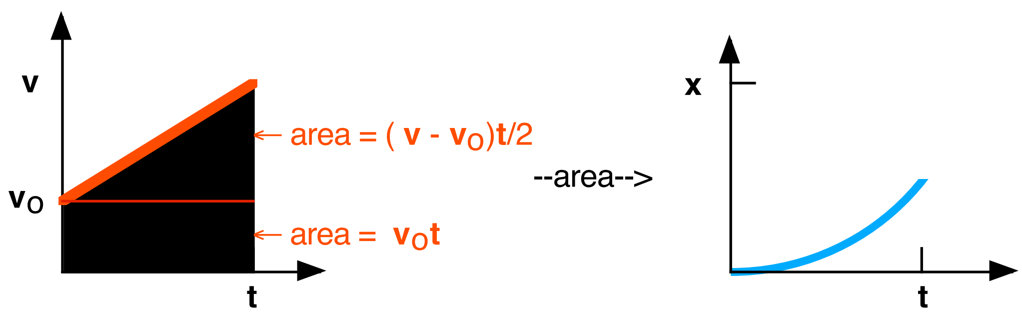
Δv = area under a *vs* $t = at$

$\Rightarrow \quad v - v_o = at$

$\Rightarrow \quad v = v_o + at$ (1)

Eq. (1) shows that the v *vs* t curve should be a straight line with a y-intercept of v_o.

$v \rightarrow x$



Δx = area under v *vs* $t = (v - v_o)t/2 + v_o t$

$\Rightarrow \quad x - x_o = (v + v_o)t/2$

$\Rightarrow \quad x = (v + v_o)t/2$ (if $x_o = 0$) (2)

Although (2) looks like a linear equation in time (whereas the x *vs.* t is anything but linear), in fact v contains time dependence. Substituting (1) into (2) to show the explicit time-dependence gives

$x = (v_o + at + v_o)t/2 = (2v_o + at)t/2$

$\Rightarrow \quad x = v_o t + (1/2)at^2$ (3)

To confirm that the form of Eq. (3) is correct, take derivatives to obtain v and a.

Equations (1) and (2) can be written in other ways as well. For example, since the plot of v *vs.* t is linear, then the average velocity v_{av} is just $(v + v_o)/2$. Hence, Eq. (2) also can be written as

$x = v_{av}t$ (4)

Alternatively, one could invert (1) to find t,

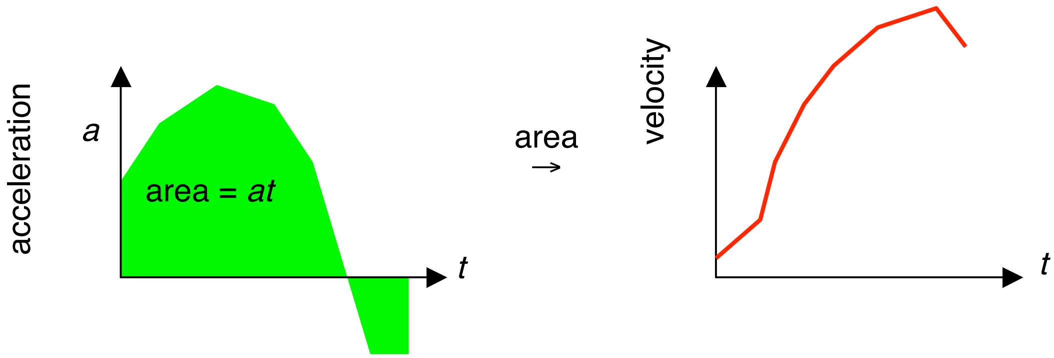
$t = (v - v_o)/a$

and substitute the result into (2) to obtain

$x = (v^2 - v_o^2) / 2a$ (5)

Variable acceleration in one dimension

Eqs. (2) - (5) only hold if the acceleration is constant. If it is not constant, the area under the *a vs. t* curve must be determined by some analytic or numerical means:

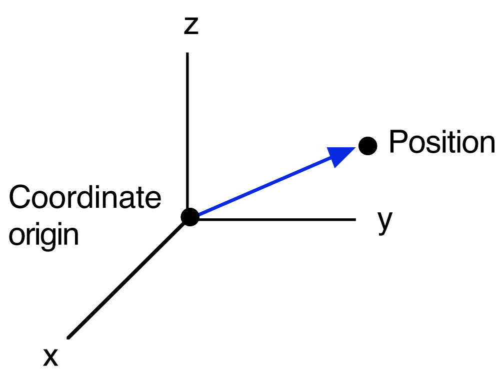


Vectors

In this course, we are concerned with motion in three dimensions, and this requires us to use vectors.

Scaler: a quantity with magnitude only, e.g. distance.

Vector: a quantity with magnitude and direction, e.g. position.

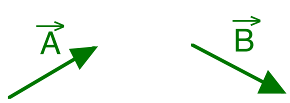


Denote position vector as $\vec{R}$. This is called the **displacement** of the object with respect to the origin.

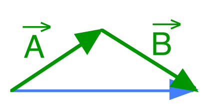
<u>Addition:</u>

The addition of vectors is not the same procedure as the addition of scalers:

Add $\vec{A}$ to $\vec{B}$



Addition rule: put tip of $\vec{A}$ to tail of $\vec{B}$, **resultant** runs from tail of $\vec{A}$ to tip of $\vec{B}$.

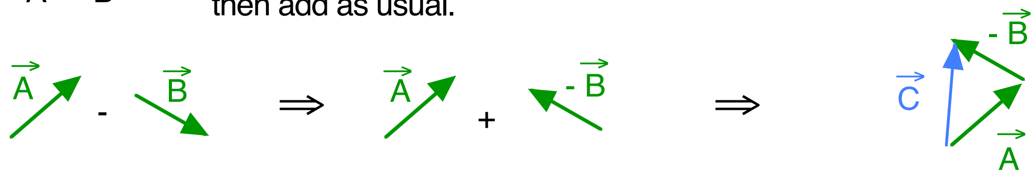


Resultant $\vec{C} = \vec{A} + \vec{B}$

Note that the order in which the vectors are added doesn't matter.

Subtraction:

$\vec{A} - \vec{B}$ Form negative of $\vec{B}$, then add as usual.



Magnitude:

The length of vector **A** is denoted by |**A**|.

Scaler times vector:

$a\vec{A} = \vec{A} + \vec{A} + \vec{A}$ ..... "a" times.

Multiplication:

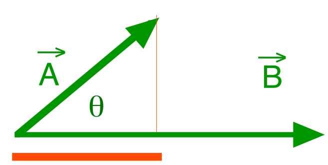
There are three products that one can form from vectors, two of which (the **dot** and **cross** product) are needed in this course. The **dot product** of two vectors is a scalar quantity, and hence the dot product also is called the **scalar product**. The notation for the dot product, and its operation, are:

$$\vec{A} \bullet \vec{B} = |\vec{A}|\,|\vec{B}| \cos\theta$$

here's the dot

this is the angle between the vectors

This operation is equivalent to taking the projection of one vector times the length of the other:



This length is A cos θ

Note that the dot product of a vector with itself is just the square of the vector's length

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{A}} = |\vec{\mathbf{A}}|\,|\vec{\mathbf{A}}| \cos(0) = A^2$$

It is often useful to represent vectors in terms of their Cartesian components:

$\mathbf{A} = (a_x, a_y, a_z)$.

Then

$\mathbf{A} + \mathbf{B} = (a_x+b_x, a_y+b_y, a_z+b_z)$

and

$\mathbf{A} \cdot \mathbf{B} = (a_xb_x, a_yb_y, a_zb_z)$.