Lecture 24 - Power and intensity

What's important:
- wave power
- intensity and decibel scale

Demonstrations: none

Power of a Wave

When we looked at the energy of an oscillating spring, we found that the total energy was

$E = (1/2)\ kA^2$ = potential energy at maximum extension.

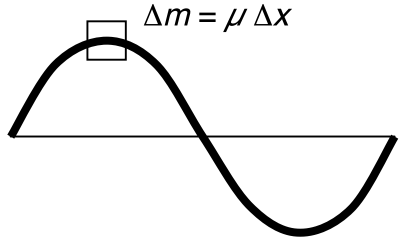
This expression can be rewritten as

$$E = (1/2)\ m\,\omega^2 A^2 \qquad (1)$$

by using

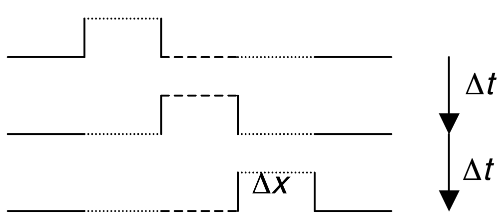
$k = m\,\omega^2$.

Although derived for a spring, Eq. (1) is valid for any oscillating system. For a wave on a string, we can use $E = 1/2\ \Delta m\,\omega^2 A^2$ for each element of mass Δm and length Δx on the string:



$$\Delta E = 1/2\ \mu\,\Delta x\,\omega^2 A^2$$

Now, consider a wave pulse where the energy is transmitted from one element Δx to the next in a time Δt



In each unit of time, the energy ΔE has moved a distance Δx down the string. $\therefore$ the power of the wave (energy per unit time) is

$$P = \Delta E / \Delta t = (1/2)\ \mu\omega^2 A^2\ \Delta x / \Delta t\ = (1/2)\ \mu\omega^2 A^2 v.$$

$$P = (1/2)\,\mu\omega^2 A^2 v.$$

Note: power goes like

ω^2 high frequency ⇒ more power

A^2 amplitude <u>squared</u>

Sound intensity

The intensity of sound is the power per unit area, or

$I = P / [area]$ (we just used A for amplitude)

The human ear can tolerate sound intensities as high as ~ 1 W/m^2, and can detect as low as 10^{-12} W/m^2. To encompass this range, we define an intensity level β by means of the logarithmic equation

$$\beta = 10 \log_{10} (I / I_o).$$

I_o is set at 10^{-12} W/m^2, the threshold of human hearing at a frequency of 1000 Hz. β is unitless in the MKSA sense, but the number generated by the equation is commonly quoted in decibels, or dB (a bel would be the same equation, but without the factor of 10).

Thus:

threshold $I = I_o$ $\beta = 10 \log (1) = 0$ dB

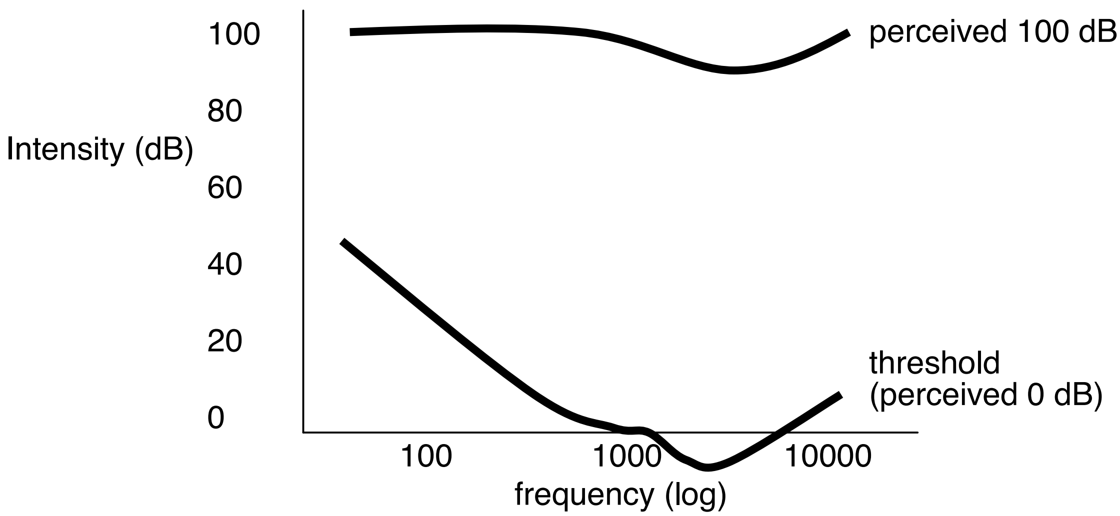
maximum $I = 1$ $\beta = 10 \log_{10} (1 / 10^{-12}) = 10 \cdot 12$

$= 120$ dB.

Some important benchmarks on the decibel scale:

Event	β (dB)
Damage to the ear	>160
Threshold of pain	120
Loud construction noise	90
Conversation	65
Whisper	20
Threshold (at 1000 hZ)	0

Now, the human ear does not respond to all frequencies equally. The response is closest to logarithmic at about 1000 Hz (i.e., a tenfold increase in I is perceived to be twice as loud at all frequencies. The ear is most efficient at about 4000 Hz. Diagramatically:



Example

An open window has dimensions of 0.5 m x 2.0 m. The sound intensity level at the window is 60 dB. How much acoustical power enters the room?

Starting with

$\beta = 10 \log_{10}(I / I_o)$

$60 = 10 \log_{10}(I / I_o)$

$\log_{10}(I / I_o) = 6$

$I = I_o \times 10^6 = 10^6 \times 10^{-12} = 10^{-6}$ W/m^2.

Using

$P = I$ [*area*]

$P = 10^{-6} \times 0.5 \times 2.0 = 10^{-6}$ W = 1 μW.

Summary of waves and oscillations

1. Basic definitions: $T = 1/f$ $\omega = 2\pi f$ (hz = Hertz = 1 s^{-1})

2. Speed: $c = \lambda f$

3. Functions: $x(t) = A \cos \omega t$ where A = amplitude
   or $A \sin(\omega t + \delta)$ in general

4. Frequency of SHM spring: $\omega = \sqrt{\frac{k}{m}}$ pendulum: $\omega = \sqrt{\frac{g}{l}}$

5. Energy: $K = 1/2\ mv^2$ $U = 1/2\ kx^2$
   $K + U = 1/2\ kA^2 = \text{constant}$

6. Doppler shift $\lambda' / \lambda = 1 \pm v/c$ (+) if away (-) if towards
   $f / f' = 1 \pm v/c$

7. Waves on a string $v = (F / \mu)^{1/2}$

   power = $P = (1/2)\ \mu A^2 \omega^2 v$

8. Travelling waves
   $y(x,t) = A \sin [2\pi (x/\lambda \pm t/T)]$ (+) to left (-) to right

9. Superposition of waves $y_{total}(x,t) = \sum_i y_i(x,t)$