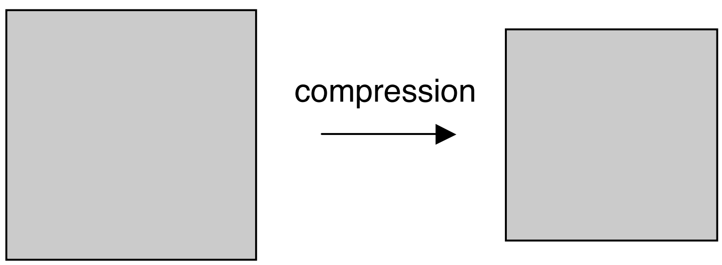
Lecture 25 - Pressure

What's important:
- pressure and density; static pressure
- Pascal's principle

Demonstrations:
- styrofoam and lead blocks; Pascal's fountain; jar+card; stool and rubber sheet; crush the can

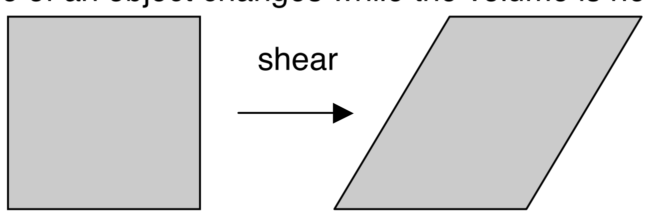
Solids and Fluids

The systems that we looked at in the first 20 lectures are either (structureless) point objects or rigid bodies. Now, let's generalize this to deformable materials. Whether a material is a solid or a fluid depends on how it responds to a deformation. We consider two types of deformation. Under compression, the volume (or density) of the object changes:



Solids and fluids both resist this kind of deformation: once the applied force is released, the material regains its original volume.

Under shear, the shape of an object changes while the volume is held fixed:

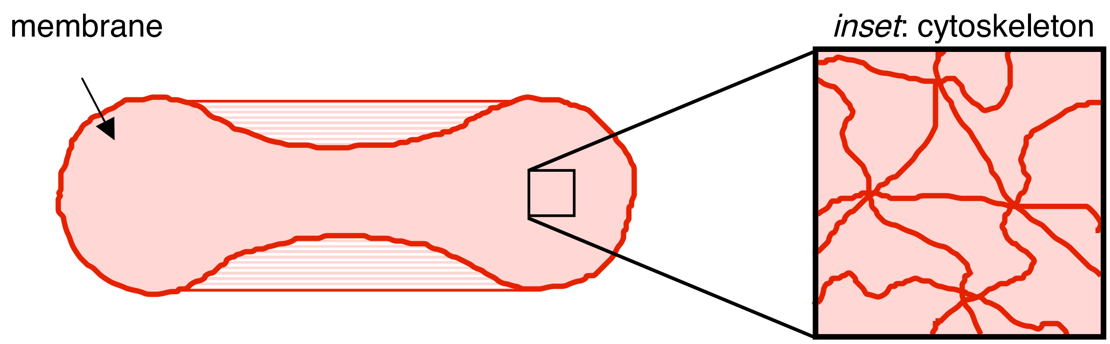


Solids can resist shear, but fluids (liquids or gases) cannot. So, if we apply a shear force to water, say by moving a spoon through a cup of coffee, the fluid responds and rearranges itself, but does not moves back to its original position once the force is no longer applied.

Biological materials

Many of the components of biological systems like the cell are either fluids, or very soft solids. The fluid parts of the cell are more than just its obvious fluid-filled interior.

Consider one of the simplest cells known - the mammalian red blood cell. For humans, the cell has the familiar biconcave shape with a diameter of about 8 microns:



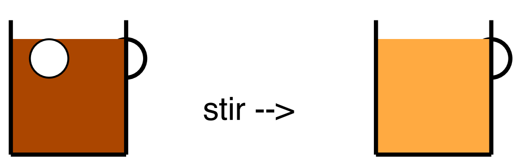
The two mechanical components of this cell are:

Plasma membrane - a **two-dimensional fluid**; the membrane acts as a barrier to control passage of ions and molecules in and out of the cell; however, in the plane of a the membrane, molecules may diffuse laterally like they would in a fluid
Membrane-associated cytoskeleton - a quasi-two-dimensional structure attached to the inside surface of the membrane, giving the cell elasticity to recover its shape after passing through a narrow capillary which may have a smaller diameter than the rest shape of the cell itself.

As a two-dimensional fluid, the plasma membrane has no shear resistance at all in the membrane plane; the red cell cytoskeleton has a resistance to compression less than a thousandth that of air at standard pressure and temperature.

Comparison of rigid and deformable bodies

The mechanics covered so far in this course provides a good description of point objects (motion characterized by v as in first few lectures) and rigid objects (fixed shapes, motion characterized by v and ω, middle lectures). Because the local speed of rotation of rigid objects obeys $v = \omega r$, only two "global" kinematic variables are required to describe the motion. With a deformable material, especially a fluid, the shape of an object may be highly distorted during motion, and one must use variables that describe the motion **locally**. For instance, if you place a drop of cream in otherwise dark coffee, and then stir, the shape of the original drop is completely lost in the ensuing motion of the coffee:



Thus, we are must describe the motion using *local* variables, such that $\mathbf{v}(\mathbf{r})$ and $\omega(\mathbf{r})$ depend upon position r in a more complex way than just $v = \omega r$. Because we describe the motion locally, then rigid body variables like mass and energy must be replaced by local variables:

Rigid body	Deformable body
$\mathbf{v}$ and ω	$\mathbf{v}(\mathbf{r})$ and $\omega(\mathbf{r})$
mass	mass density
energy	energy density

Force is replaced by force/area, or pressure, which also has units of energy density.

Density

Mass density is obtained from **mass** by the simple relation:

[*density*] $\equiv \rho \equiv$ [*mass*] / [*volume*] (ρ = Greek letter rho)

Densities of some common materials:

material	density (kg/m^3)
air	1.29
styrofoam	100
water	1000
aluminum	2700
iron	7860
gold	19,300

Demo: styrofoam and lead blocks of equal volume

Note the density of air. At a kg per cubic metre, it doesn't seem like much - but it is equal to a large box of cookies per cubic meter. But when we think about the mass of air in the entire atmosphere pressing down on a flat table-top of area 1 m^2, it's impressive. Suppose that the atmosphere has a density of 1.29 kg/m^3 up to a height of 5 km (like the highest mountain in the Rockies). The mass of a box of air 5 km high and 1 m^2 in cross sectional area is

mass = $\rho V = 5 \times 10^3 \times 1 \times 1.29 \sim 6000$ kg = 6 tonnes.

That's impressive, although the correct calculation would give 10 tonnes rather than 6. To see the effects of this mass, let's do an experiment where we pump the air out of a can, so that its sides are subject just to atmospheric pressure.

Demo: pump the air from within a 1 gallon tin can and observe how it is crushed by atmospheric pressure.

Energy density and pressure

Now, if we replace mass by mass density ρ, so we must replace energy by energy density, or energy per unit volume:

[*energy*] / [*volume*] = [*force*] • [*distance*] / [*volume*] = [*force*] / [*area*].

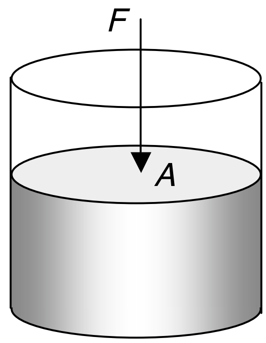
Thus, energy density has the same units as force per unit area, which we call pressure:

pressure = force per unit area

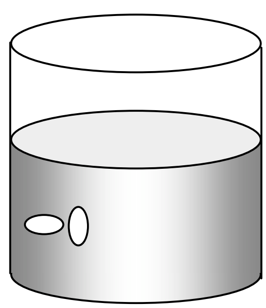
or

$P = F / A$

The external pressure experienced by a fluid equals the force on its surface divided by the surface area



The pressure isn't felt just on the surface of a fluid, but is transmitted throughout its volume (through action-reaction couplets over all the interior fluid elements). Within a **fluid** (not solids!), the pressure at a position is independent of the orientation of the local surface. For example, the pressure on the disks in the diagram is independent of the orientation of the disks



Demo: Pascal's fountain (water-filled flask with small holes in an arc, force applied by a rubber piston). When the plunger is pushed, water escapes through the holes at roughly equal rates in all directions - it doesn't escape faster though a hole opposite the plunger, and slower through a hole off to the side.

Atmospheric pressure (due to air) varies somewhat according to the weather, but an average value is

$P_{atmos} \sim 1.01 \times 10^5$ N/m^2 $\equiv$ 1.0 atmosphere

While we're talking about units, the Pascal = Pa is defined as 1 N/m^2.

Often, we are interested in pressures with the background pressure of the atmosphere removed. This is referred to as the gauge pressure:

$P_{gauge} = P - P_{atmos}$.

Demo: stool lifted by a sheet of rubber lying flat on its surface.

Demo: fill a jar with water and place an index card across its mouth. When inverted, atmospheric pressure holds the card in place.