**Lecture 27 - Buoyancy and fluid flow**

*What's important:*
- Archimedes principle
- streamline flow

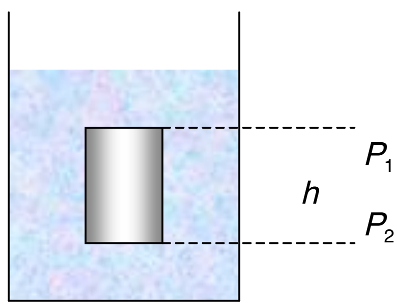
*Demonstrations:*
- floating objects; scales and bowling balls

**Archimedes Principle**

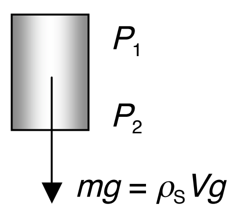
In the previous lecture, we showed that the pressure at two different heights in a fluid are related by

$$P_2 - P_1 = \rho g h$$

where $\rho$ is the density. This means that an object placed in a fluid experiences a different pressure on its top and bottom sides:




Here, we take $h > 0$ so that $P_2 > P_1$. The pressures at all points with the same height, as indicated by the dashed lines, are equal, meaning that the object experiences a pressure difference




The net force from the surrounding fluid on the object is equal to the upward force,

$$(P_2 - P_1)A = \rho_L g h A = \rho_L g V \qquad \text{(assuming a cylinder with } V = Ah\text{)}$$

This force applies to any object placed in the fluid. Further, it is independent of the shape of the object, as can be verified with a little thought.

Thus, we define the buoyant force $B$ as

$$B = \rho_L g V.$$

The effective weight of the body when it is immersed is then

$$w_{\text{eff}} = w - B$$
$$= \rho_s g V - \rho_L g V$$
$$= \rho_s g V (1 - \rho_L/\rho_s)$$
$$= w\,(1 - \rho_L/\rho_s).$$

In other words, the weight is reduced by the ratio of the densities.

If $\rho_L < \rho_s$, then $w_{\text{eff}} > 0$ and the object sinks: the net force is still negative.

But if $\rho_L > \rho_s$, then $w_{\text{eff}} < 0$ and the object rises until it floats, where the effective weight vanishes. When an object floats, the volume of the fluid displaced is less than the volume of the object (careful with the "volume" of a boat), although the masses are equal:

$$\rho_L V_L = m_{\text{object}}$$

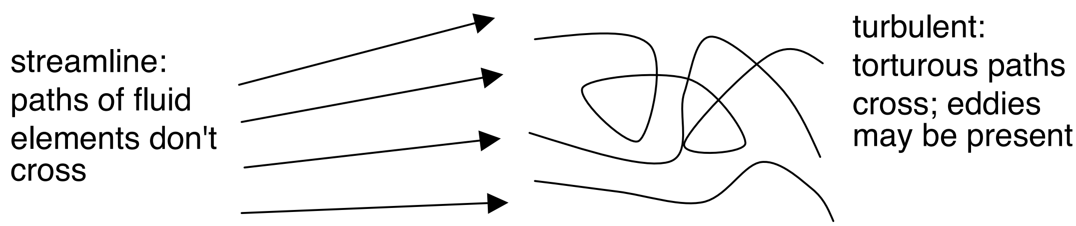
Archimedes' Principle states the mathematical result in words:

"If a body is wholly or partially immersed in a fluid, it experiences an upward thrust equal to the weight of the fluid displaced, and this upward thrust acts through the centre of gravity of the displaced fluid."

**Streamline flow**

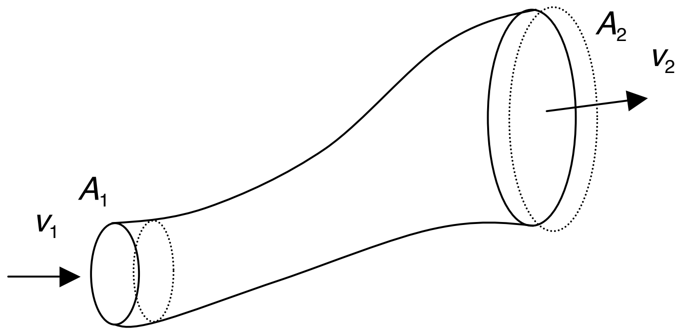
We've now described how stationary fluids behave, including the distribution of pressure, either because of gravity or because of an applied force (Pascal's principle). Let's now move on to moving fluids.

We can categorize the motion of fluids as either streamline or turbulent:




We now derive an equation for the motion of a fluid under streamline flow.

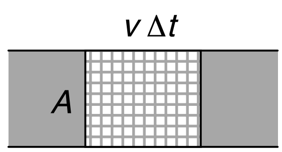
Assume for the moment that the density of the fluid can vary locally.




The fluid enters at the left (end #1) with velocity $v_1$ and exits to the right with velocity $v_2$.

The lines indicate streamlines, the paths taken by a ring of area $A_1$ at end #1.

Now, the volume swept out by the ring at end #1 in time $\Delta t$ equals the area $A_1$ times the distance covered in time $\Delta t$, namely $v_1 \Delta t$. That is




$$[volume]_1 = A_1 \; v_1 \Delta t$$

This means that the mass passing through end #1 is

$$\Delta m_1 = [mass] = [density] \bullet [volume] = \rho_1 \; A_1 \; v_1 \Delta t$$

The same argument holds at end #2,

$$\Delta m_2 = \rho_2 \; A_2 \; v_2 \Delta t.$$

But these masses must be equal (streamlines don't cross), so

$$\rho_1 \; A_1 \; v_1 = \rho_2 \; A_2 \; v_2 \qquad \textit{Equation of continuity}$$

Further, if the fluid is incompressible (liquids are difficult to compress, gases are not), then $\rho_1 = \rho_2$, and we have

$$A_1 \; v_1 = A_2 \; v_2$$

This equation says that the narrower the channel, the faster the flow (or conversely, still water runs deep). Common example: nozzle of a garden hose – narrow = fast.

NOTE: if the velocity is not uniform across the area elements $A$, then we define

[*average velocity of flow*] = [*volume rate of flow*] / [*cross section*]