Lecture 28 - Bernoulli's equation

What's important:

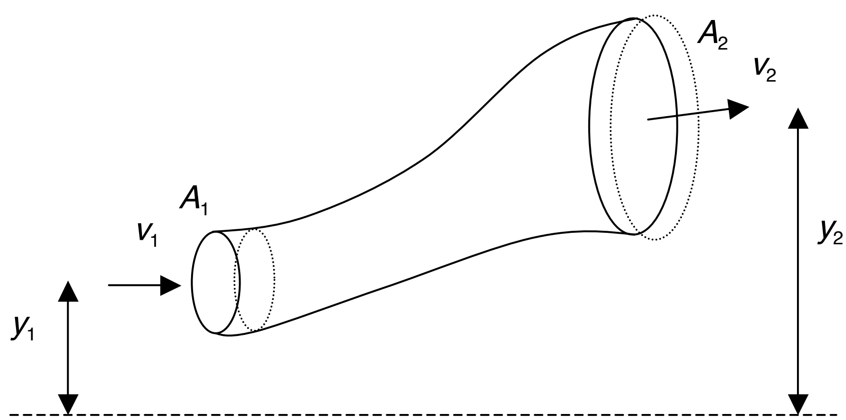
- Bernoulli's equation
- Venturi tube

Demonstrations:

- Venturi tube; misc. air flow demonstrations (funnels, vacuum cleaners, aspirators)

Bernoulli's equation

Let's specialize to the case of an incompressible fluid (ρ = constant) but now include the effects of gravity and kinetic energy. We return to the diagram for streamline flow that we introduced before:



At end #1, the total mechanical energy of the volume element is

$$E_1 = 1/2\ \Delta m_1\ v_1^2 + \Delta m_1 g y_1$$

The same applies at end #2:

$$E_2 = 1/2\ \Delta m_2\ v_2^2 + \Delta m_2 g y_2$$

If the flow is steady, there is no change in the mechanical energy at a location as a function of time. Further, since mass is conserved in the flow, then

$$\Delta m_1 = \Delta m_2 = \Delta m,$$

and we have

$$E_2 - E_1 = 1/2\ \Delta m\ v_2^2 + \Delta m g y_2 - 1/2\ \Delta m\ v_1^2 - \Delta m g y_1.$$

But this difference in energy must come from the work being applied to, or done by, the fluid. Again, taking the density ρ to be constant.

At end #1:

work done on the fluid = W_1 = [*force*] • [*distance*]

$= (P_1 A_1) \cdot (v_1 \Delta t)$

$= P_1 (\rho A_1 v_1 \Delta t) / \rho$.

But $\rho A_1 v_1 \Delta t$ is just the mass Δm entering at end #1, so

$$W_1 = P_1 \Delta m / \rho.$$

The same argument applies at end #2, but now the work is being done by the system and hence the sign is reversed.

$$W_2 = -P_2 \Delta m / \rho.$$

Thus,

$$W_1 + W_2 = (P_1 - P_2)\Delta m / \rho.$$

But the work supplies the energy difference $E_2 - E_1$, which is

$$E_2 - E_1 = \Delta m\{1/2 (v_2^2 - v_1^2) + g(y_2 - y_1) \}$$

Thus

$$(P_1 - P_2)\Delta m / \rho = \Delta m\{1/2 (v_2^2 - v_1^2) + g(y_2 - y_1) \}$$

$$P_1 - P_2 = \rho\{1/2 (v_2^2 - v_1^2) + g(y_2 - y_1) \}$$

or

$$P_1 + \rho v_1^2/2 + \rho g y_1 = P_2 + \rho v_2^2/2 + \rho g y_2$$ *Bernoulli's equation*

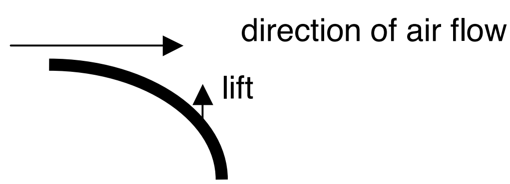
Consequences

One thing to note immediately about Bernoulli's equation is the situation where the height is unchanging: $y_1 = y_2$. Then

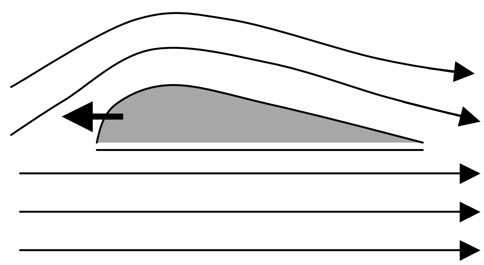
$$P_1 + \rho v_1^2/2 = P_2 + \rho v_2^2/2.$$

This says that pressure decreases with velocity. This effect underlies the operation of airplane wings and many other devices, also responsible for tornadoes ripping the roofs off houses.

demo: blow horizontally over a piece of paper



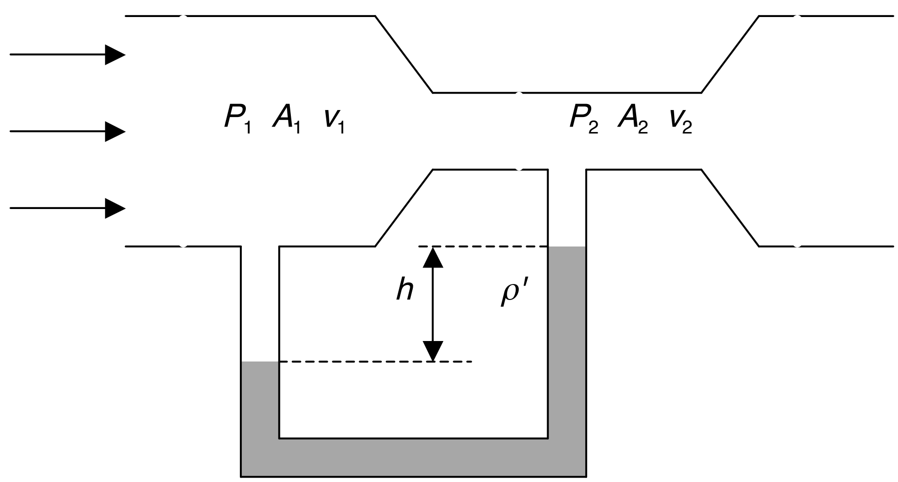
operation of wing



pressure is lower on the top side of the wing because the air is moving faster
---> pressure difference gives the wing lift

Venturi tube

The Venturi tube (and its variants) provide a means of measuring the speed of a fluid through pressure differences; useful on pipes and slow planes.



Bernoulli's equation without gravity

$$P_1 + \rho v_1^2/2 = P_2 + \rho v_2^2/2$$

or

$$P_1 - P_2 = \rho\,(v_2^2 - v_1^2)\,/\,2$$

We want to determine v_1; assume an incompressible fluid to get rid of v_2:

$$v_1 A_1 = v_2 A_2$$

or

$$v_2\,/v_1 = A_1/A_2$$

Then

$$P_1 - P_2 = \rho\,(v_2^2 - v_1^2)\,/\,2$$
$$= \rho\, v_1^2\,(v_2^2/v_1^2 - 1)\,/\,2$$

$= \rho\, v_1^2\, (A_1^2/A_2^2 - 1) / 2$

We can invert this equation to solve for v_1:

$$v_1^2 = 2(P_1 - P_2) / \rho(A_1^2/A_2^2 - 1).$$

In the apparatus, the pressure difference is

$$P_1 - P_2 = \rho' g h$$

$$v_1^2 = 2(\rho'/\rho) g h / (A_1^2/A_2^2 - 1).$$

Now, almost everything appearing on the right-hand side is a property of the apparatus, so we can write

$$v_1^2 = K\, h$$

where

$$K = 2(\rho'/\rho) g / (A_1^2/A_2^2 - 1).$$