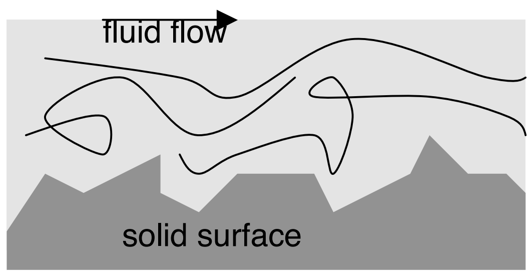
Lecture 29 - Viscosity

What's important:
- viscosity
- Poiseuille's law
- Stokes' law

Demo: dissipation in flow through a tube

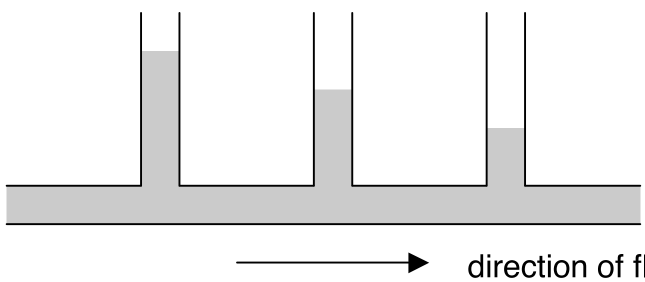
Viscosity

The fluids that we have described in the previous lectures are ideal in the sense that there are no dissipative losses (or friction) in their motion. But we know that the boundary of a fluid and a wall must be rough, and exert drag on a flowing fluid just like friction does between solid surfaces pushed into contact by a force.

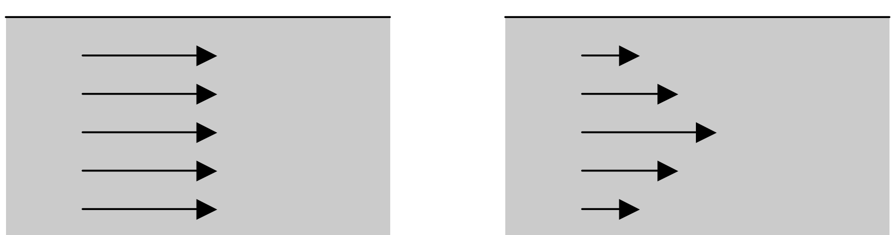


The magnitude of the drag force on a fluid is characterized by its viscosity η.

Demo: The effects of dissipation show up as a pressure gradient at constant height *y*



Within a tube, there is a velocity gradient in the presence of viscosity:



How do we characterize this? The volume rate of flow *Q* is defined as

$Q \equiv \Delta V / \Delta t$,
that is, the rate of change of volume.

From before, we saw that the change in volume in a given time Δt is
$\Delta V = vA\,\Delta t$,

where v is the fluid velocity and A is the cross sectional area of the fluid element. If v is not a constant, this equation still applies, but now we must use the average velocity v_{av},
$\Delta V \equiv v_{av}\,A\,\Delta t$.
This gives
$Q = v_{av}A$.

Poiseuille's law

Poiseuille's law describes laminar (non-turbulent) fluid flow through a cylinder of radius R
$Q = (\pi R^4 / 8\eta) \cdot (\Delta P / L)$

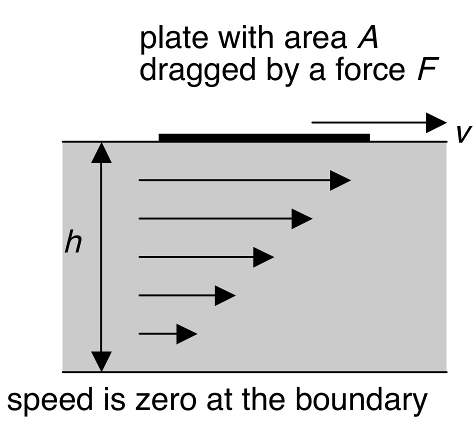
where ΔP is the pressure drop over the length L of the tube. Note that the flow increases like R^4, a rather large power of R.

We introduced the viscosity η some time back; it has units of J•s/m^2 $\equiv$ 10 P, where P is a Poise. Again, some sample values:

Fluid	η (kg/m•sec at 20 ^{o}C)
Air	1.8×10^{-5}
Water	1.0×10^{-3}
Mercury	1.56×10^{-3}
Olive oil	0.084
Glycerine	1.34
Glucose	10^{13}

mixtures: η of blood = 2.7×10^{-3} kg/ m•s

We showed earlier how to obtain η from the terminal speed of an object falling under gravity. An alternate approach uses:



$F / A = \eta v / h$

Example (from Walker): blood flow in the pulmonary artery, which carries blood from the heart to the lungs). The pulmonary artery is 8.5 cm long, 2.4 mm in radius and supports a pressure difference along its length of 450 Pa. What is the average speed of the blood in the artery?

From Poiseuille's law,

$$Q = \pi R^4 \, \Delta P / 8\eta L \qquad + \qquad Q = v_{av} A$$

$$v_{av} A = \pi R^4 \, \Delta P / 8\eta L$$

Dividing by $A = \pi R^2$, we have

$$v_{av} = R^2 \, \Delta P / 8\eta L$$

Upon numerical substitution

$$v_{av} = (2.4 \times 10^{-3})^2 \, 450 \, / \, (8 \cdot 2.7 \times 10^{-3} \cdot 8.5 \times 10^{-2})$$

$$= 1.4 \text{ m/s.}$$

Stokes' Law

Lastly, we just recap that for a spherical object moving slowly through a fluid (streamline, not turbulent), the drag force is given by

$$F_{\mathrm{DRAG}} = 6\pi\eta R v.$$

Reynolds number

A rule of thumb for classifying flow about an object of length ℓ is the so-called Reynolds number, a dimensionless quantity given by

$$\mathrm{Re} = \rho \, v \, \ell \, / \, \eta$$

where v is the velocity of the object, ρ is the density of the medium and η is its viscosity. Flow is streamline at small Re and turbulent at large Re; the cross-over between the regions depends on the geometry of the environment, and is in the 10-100 range. For a cell moving in a medium with a few times the viscosity of water, say 3×10^{-3} kg/m•s, one has

$$\mathrm{Re} = 10^3 \text{ (kg/m}^3\text{) } 10^{-6} \text{ (m/s) } 10^{-6} \text{ (m) } / \, 3\mathrm{x}10^{-3} \text{ (kg/m}\bullet\text{s)}$$

$$= 3\mathrm{x}10^{-7}.$$

Well, this is certainly much less than 1.