Lecture 34 - Physical properties of matter

What's important:

- thermal expansion
- elastic moduli

Demonstrations:

- heated wire; ball and ring; rulers in liquid N_2

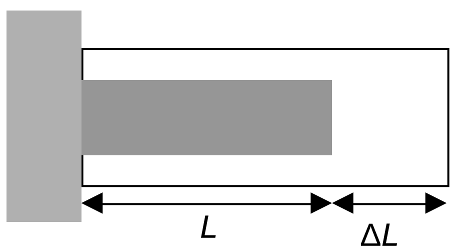
Text: Walker, Secs. 16.3, 17.3
Problems:

The shape of an object may change in response to a change in temperature or to an applied stress. We treat each of these phenomena in turn.

Thermal expansion

We saw in our derivation of the ideal gas law that the speed of molecules increases with temperature, with the result that they exert more pressure on their container. This phenomena applies in general, and for solids, implies that materials usually expand when they are heated (although there are known counterexamples).

General observation is that the relative expansion is proportional to the change in temperature



In symbols:

$$\Delta L \,/\, L = \alpha \, \Delta T,$$

where L is the original length, ΔL is the change in length and ΔT is the change in temperature.

The proportionality constant α is called the coefficient of linear expansion, having dimensions of [*temperature*]$^{-1}$. Some typical values

Material	α ($C^{o\,-1}$)
steel	12×10^{-6}
brick	9×10^{-6}
rubber	80×10^{-6}

Demo: heated wire

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Example

A suspension bridge has a span of 100 m. If the bridge deck were made of a single sheet of steel, what would be the change in length from -10 ^oC to +30 ^oC?

Ans: $\Delta T = 40$ C and $\alpha = 12 \times 10^{-6}$ C^{-1}

--> $\Delta L / L = 12 \times 10^{-6} \times 40 = 4.8 \times 10^{-4}$

For a 100 m span, this corresponds to

$\Delta L = 4.8 \times 10^{-4} L = 4.8 \times 10^{-4} \times 100 = 4.8 \times 10^{-2}$ m

While 5 cm doesn't seem like much, the bridge design must take it into account to avoid buckling.

Demo: ball and ring dipped in liquid nitrogen
Demo: ruler in liquid nitrogen

Volume expansion

If a material expands in all directions, then its volume expands as well. In some cases, we are interested in the volume expansion, rather than the linear expansion of a material: for example, we may have a fluid in a sealed container and want to know how the volume increases with temperature to make sure that the container can handle the change. Then, we parametrize:

$$\Delta V / V = \beta \, \Delta T$$

where β is the coefficient of volume expansion.

We can find an approximate expression for β through the following argument:

· say we have a cube of side L, such that the volume changes from
 $V = L^3$ to $V = (L + \Delta L)^3$

· then $\Delta V / V = [(L + \Delta L)^3 - L^3] / L^3 = [L^3 + 3L^2 \bullet \Delta L + 3L \bullet (\Delta L)^2 + (\Delta L)^3 - L^3]/L^3$
 $= [3L^2 \bullet \Delta L + 3L \bullet (\Delta L)^2 + (\Delta L)^3]/L^3$

· now, if ΔL is small compared to L, then only the first term in this expansion is important, and
 $\Delta V / V = 3L^2 \bullet \Delta L / L^3 = 3 (\Delta L / L)$

· but we know that
 $\Delta L / L = \alpha \, \Delta T$,

so
 $\Delta V / V = 3\alpha \, \Delta T$.

Comparing with our expression for the volume expansion coefficient, we see
 $\beta = 3\alpha$.

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Typical values: fluids tend to expand more than solids

Material	β ($C^{o\,-1}$)
ethyl alcohol	1.1×10^{-3}
glycerine	5.1×10^{-4}
water	2×10^{-4}

Counter-example: H_2O is most dense at +4 C, which is the temperature in lake bottoms in the winter (or, why lakes don't freeze solid).

Demo: water thermometer

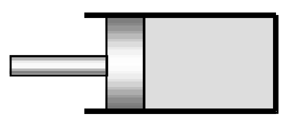
Stress and strain

A force is applied over a region of the surface of an extended object, rather than a specific point on its surface. Thus, we talk about quantities like pressure when discussing fluids, *etc*. Now we extend the idea of pressure to take into account the direction of the applied force with respect to the orientation of the surface.

First, we define a general quantity called stress, which is the force per unit area:

[*stress*] = [*force*] / [*area*]

In situation like



where a fluid (say, a gas) is confined by a cylinder,

[*stress*] = [*pressure*].

Isn't stress always pressure? Certainly pressure is an example of stress, but there are others, where the applied force is not perpendicular to the surface.

The **strain** characterizes the deformation of an object in response to a stress. Strain is dimensionless, and equals the *relative* change in shape of an object. The relative change in volume is **one example** of the **strain**:

$\Delta V / V$ = [*strain*],

The general relation between stress and strain was proposed by Robert Hooke

[*stress*]   [*strain*]

which applies at small strains and can be used to obtain $F = kx$ for springs.

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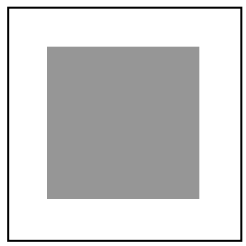
Elastic moduli

For isotropic materials (same in all directions), there are two fundamental elastic moduli, the bulk modulus B and the shear modulus μ.

Bulk modulus

This modulus characterizes the change in the relative volume induced by an isotropic pressure P

$$\Delta P = B\,(\Delta V / V)$$

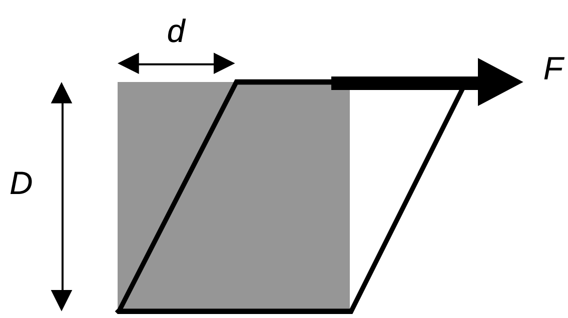


Because the strain is dimensionless, B has the same units as pressure; *i.e.*, it has units of energy density. Examples of bulk moduli

material	B (J/m^3)
tungsten	20×10^{10}
aluminum	7×10^{10}
polystyrene	0.5×10^{10}
water	0.22×10^{10}

Shear modulus

For this deformation, the stress may be applied parallel to a surface



Here, the shear strain is d / D, and the equation reads

$$F / A = \mu\, d / D.$$

For solids, the shear modulus is often about half of the bulk modulus. For fluids, the shear modulus vanishes.

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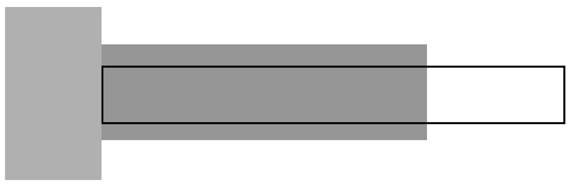
Some examples:

material	μ (J/m^3)
tungsten	15×10^{10}
bone	8×10^{10}
lead	0.54×10^{10}
fluids	0

Young's modulus

The Young's modulus arises for uniaxial stress



$$F/A = Y(\Delta L/L)$$

Y can be derived from B and μ. Typical values:

material	Y (J/m^3)
tungsten	36×10^{10}
pyrex glass	6.2×10^{10}
bone (tension)	1.6×10^{10}
nylon	0.37×10^{10}
polystyrene	0.14×10^{10}
proteins	0.05×10^{10}

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