Lecture 4 - Centripetal acceleration

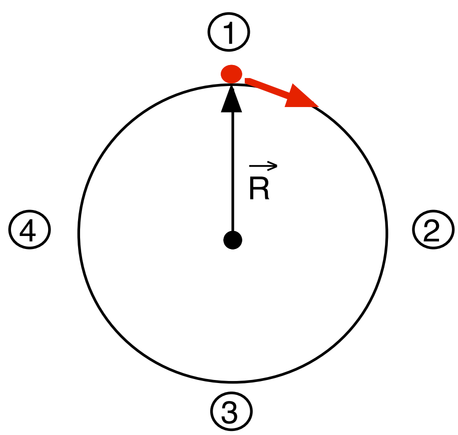
What's important:
- uniform circular motion
- centripetal acceleration

Demo: Tennis ball on string

This material does not take a full lecture; combine with adjacent lectures.

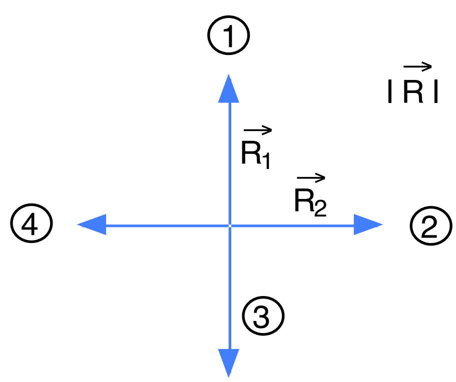
Circular Motion

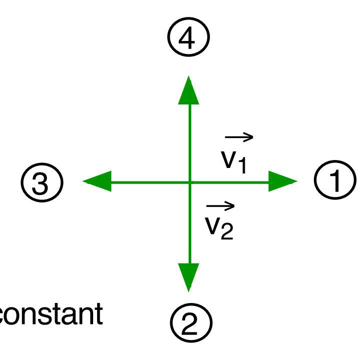
As a simple, but often confusing, application of vectors, we consider circular motion. In the diagram, an object moves in a circular path with radius **R**, in a clockwise direction (as indicate by the red arrow):



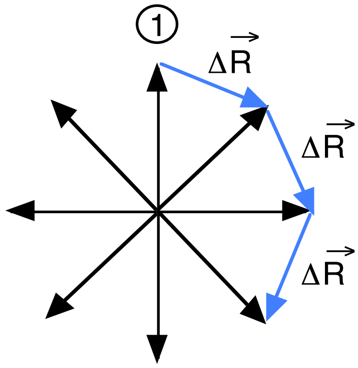
$|\vec{R}|$ = radius (fixed)

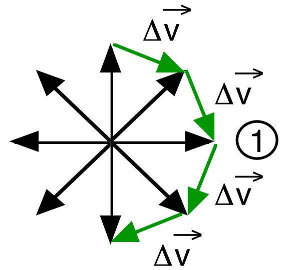
Let's look at the displacement and velocity vectors at four different positions, labelled 1 ... 4 in the diagram.





We can fill in the various intermediate positions to see how the **change in position** and **change in velocity** themselves change in time.





During the period **T** for one complete revolution,

Total distance covered $= \sum |\Delta \mathbf{R}|$
$= 2\pi \mathbf{R}$

$\therefore$ speed $= |\vec{v}|$ = distance / time
$= 2\pi R / \mathbf{T}$

Total change in velocity $= \sum |\Delta \mathbf{v}|$
$= 2\pi v$

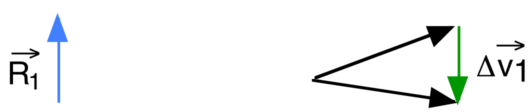
$\therefore$ acceleration $= 2\pi\ \mathbf{v} / \mathbf{T}$

$$= 2\pi \mathbf{v} \left(\frac{\mathbf{v}}{2\pi \mathbf{R}} \right) = \frac{\mathbf{v}^2}{\mathbf{R}}$$

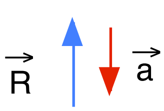
At first sight, this may be a surprising result: the speed is constant, but there is an acceleration $\mathbf{a} = \mathbf{v}^2 / \mathbf{R}$. Of course, even though the speed is not changing, the velocity does change because it is changing direction.

Demo: use tennis ball on a string to illustrate how **v** *varies inversely with* **R**.

Finally, note that $\Delta\vec{\mathbf{v}}$ is in the opposite direction to $\vec{\mathbf{R}}$.



Further, since $\vec{\mathbf{a}}$ is parallel to $\Delta\vec{\mathbf{v}}$, then $\vec{\mathbf{a}}$ must be in the opposite direction to $\vec{\mathbf{R}}$ as well:



We call $\vec{\mathbf{a}}$ the centripetal (or centre - seeking) acceleration.