Lecture 5 - Dynamics

What's important:
- Newton's three laws of dynamics

Demonstrations:
- force constant of a spring
- scales for examples
- clear plastic sheet for force on hand
- force constant of an elastic rises with temperature

Dynamics

Kinematics permits us to describe the motion of particles expressed in terms of **x** and its rates of change **v** and **a**. **Dynamics** relates the motion of particles to the forces between them. Together, they constitute **mechanics**. In these lectures, we examine **Newtonian classical** mechanics:
- formulated by Newton (1642-1727) in terms of **x**, **v** and **a**.
- classical --> speeds small compared to the speed of light.

The first law tells us what happens if nothing happens:

Newton's First Law

An object continues in its initial state of rest or motion with uniform velocity unless it is acted upon by an unbalanced force.

Physical intuition: suppose we are in a spaceship moving with respect to the Earth but far away from any planets, etc. Then if the windows of the spaceship were covered over, we could not tell from the inside of the spaceship whether it was moving [this is not true when it is accelerating!]. So, Newton's first law does two things:
- it puts "at rest" or "motion with uniform velocity" on the same footing
- it says that nothing happens to this motion unless the object is acted upon by an unbalanced force.

This is *very* different from Aristotle's view of the friction-dominated world, which held sway for almost 2 millenia, namely "everything stops unless there is a force to keep it moving".

What does a force do to an object? This is **Newton's Second Law**.

The acceleration a of an object subject to an unbalanced force $\mathbf{F}_{net}$ is directly proportional to $\mathbf{F}_{net}$ and inversely proportional to its mass *m*:

$$\mathbf{a} = \mathbf{F}_{net} / m \qquad \text{or} \qquad \mathbf{F}_{net} = m\,\mathbf{a}.$$

Notes:

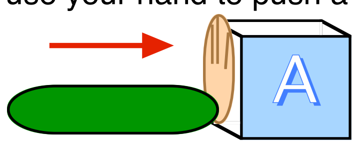
- This is a vector equation and applies component by component:

  $a_x = F_x / m \quad a_y = F_y / m \quad a_z = F_z / m.$
- **a** is parallel to $\mathbf{F}_{net}$.
- $\mathbf{F}_{net}$ is the **vector** sum of all applied forces: $\mathbf{F}_{net} = \Sigma_i \mathbf{f}_i$.

Newton's Third Law deals with the interaction of the body delivering the force and the body receiving the force:

Forces always occur in pairs. If object A exerts a force $\vec{F}$ on object B, then object B exerts a force $-\vec{F}$ on object A. For every action there is an equal and opposite reaction.

Consider an example: use your hand to push a block across a table



We see the motion of the block and think that the only force present is the one acting on the block. But look at the hand: it is slightly flattened up against the block and the reason why it is flattened is the force which the block is exerting on the hand (*demo*).

Some common forces

$1/r^2$ *force* (Newton's law of gravity and Coulomb's law)

$F = Gm_1m_2 / r^2$ gravity

$F = kQ_1Q_2 / r^2$ electrostatics

On the surface of the Earth, $m_1 = m_{Earth}$ $m_2 = m$ (of object) $r = R_{Earth}$

$$F = (Gm_{Earth} / R_{Earth}^2)\ m$$

Using

$G = 6.67 \times 10^{-11}$ Nm^2/kg^2 $m_{Earth} = 5.98 \times 10^{24}$ kg $R_{Earth} = 6.37 \times 10^6$ m

we obtain

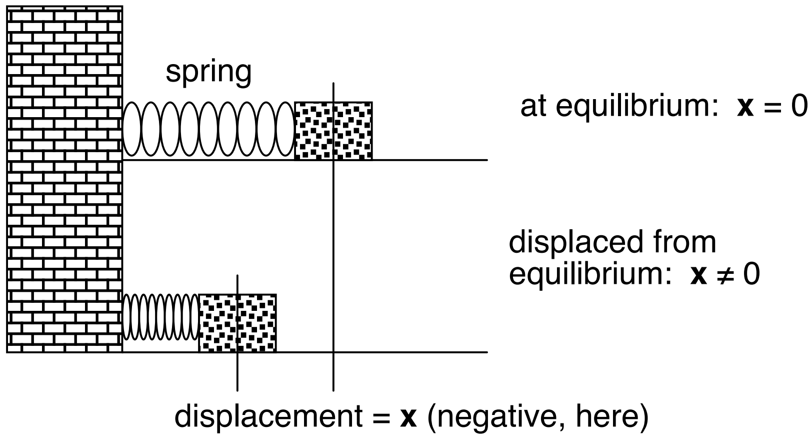
$$(Gm_{Earth} / R_{Earth}^2) = 9.8 \text{ m/s}^2 = g$$

Force proportional to distance

The force associated with springs and elastic bands (in fact, most materials) at small deformations is called Hooke's law:

$$\mathbf{F} = -k\,\mathbf{x},$$

where **x** is the (vector) displacement from equilibrium.

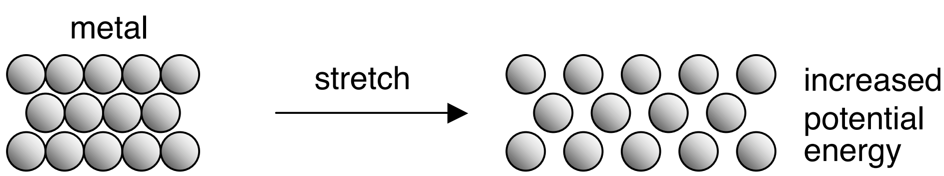


Demo: masses m, 2m, 3m, 4m on identical springs show linearity of force vs extension

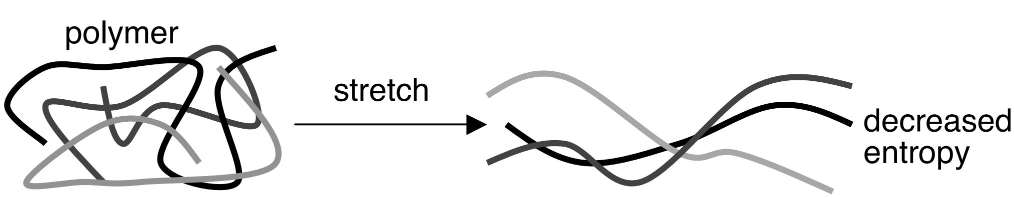
Demo: determine spring constant from plot of mg *vs* **x**.

Origin of elasticity

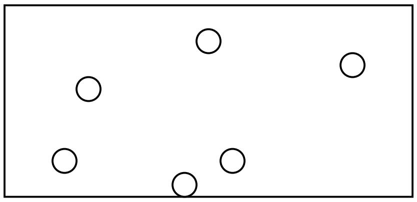
Elasticity can be viewed as having contributions from a change in energy or a change in entropy. For example, when a lattice of close-packed atoms, as in a solid metal, are subject to an external tension, they move from their equilibrium position, resulting in an increase in their potential energy:



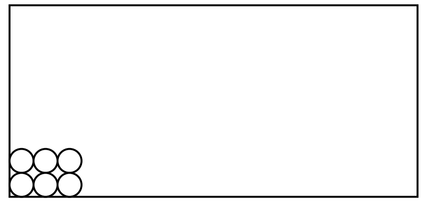
This is in contrast with the behaviour of a polymer (a network of proteins, for example), which may not be close-packed and have a lot of freedom of movement:



In the language of thermodynamics, the stretch has decreased the entropy of the chain network. We'll return to a discussion of entropy later in the course, but for now we define it as a measure of the number of configurations available to a system. For example, a molecular gas has high entropy when it can explore the entire volume of its container (lots of configurations) but lower entropy when it is forced into one corner of the container.



high entropy



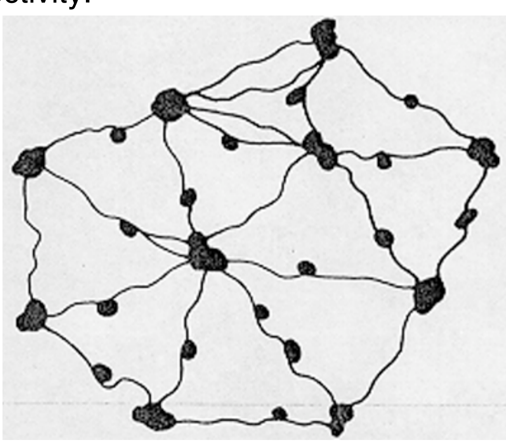
low entropy

We know from everyday experience that work is required to compress a gas into a smaller volume – i.e., work is required to decrease the entropy of the gas. So too with the polymer network: work is required to stretch the network because its entropy is being reduced.

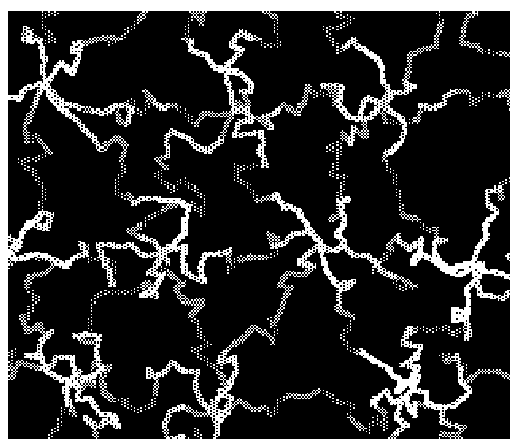
Entropy becomes more important compared to energy as the temperature of a system rises. For a polymer network, this means that it becomes more difficult to stretch the network at higher temperatures. In other words, the **force constant** of the network rises with temperature.

Demo: a stretched elastic band shrinks when heated.

The protein scaffolding in a red blood cell has been a highly-studied system in cell biology. It's a two-dimensional network lining the cell membrane with six-fold connectivity.



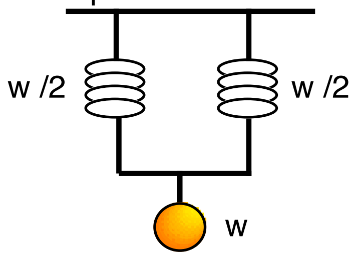
drawing from a microscope

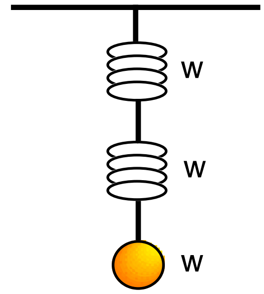


computer simulation

The thin filaments are the protein *spectrin* and they have a spring constant of ~10^{-5} N/m.

Springs in series and parallel





Construct free-body diagrams for each. Show by demonstration why the above weights are observed.