Demonstrations:
•microwave source and detector
•box of marbles
Text: Mod. Phys. 9.A, 9.B, 9.C
Problems: 3, 4, 5 from Ch. 9

What's important:
•$\mathbf{k_B T}$ is a measure of the average kinetic energy of a particle
•number density and energy density of photon gas
•Big Bang model

Temperature and energy

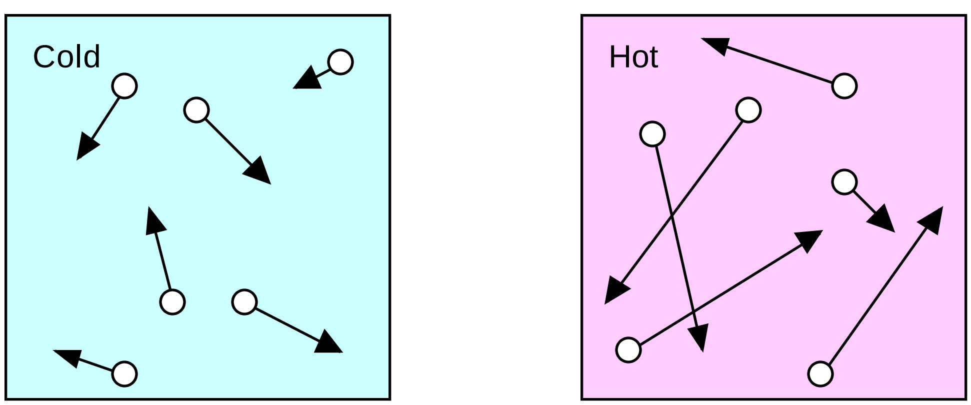
It has been known for almost two centuries that gases at low density obey the ideal gas equation,

$$\mathbf{PV} \quad \mathbf{T}, \tag{1}$$

where the gas in question is subject to a pressure **P**, occupies a volume **V** and has an absolute temperature **T**. Pressure has units of energy density, and the left hand side of Eq. (1) is proportional to (but not exactly equal to!) the kinetic energy of the gas, so that the kinetic energy of an atom or molecule in the gas is proportional to the temperature:

$$[\textit{kinetic energy}] \quad \mathbf{T}.$$

Consider the following problem: we place a number of marbles in a box and shake the box back and forth continuously. The marbles start to move, colliding with each other and with the sides of the box.



(a) (b)

If we could determine the kinetic energy of every marble at every instant in time we would find:

•not all marbles have the same kinetic energy at the same time. Some marbles are moving slowly, some fast, so the marbles have a *distribution* of kinetic energies.

•a given marble exchanges energy with its surroundings through collisions, so the kinetic energy of each marble changes with time.

The functional form of the kinetic energy distribution was found by Maxwell in 1859, and the time evolution of such distributions was described by Boltzmann in 1872. For an ideal gas of marbles in a three-dimensional box (no internal structure to the marbles) the average kinetic energy per particle is related to the temperature of the system by

$$[\textit{average kinetic energy per particle}] = (3/2)\, \mathbf{k}_B \mathbf{T} \qquad (2)$$

$$\mathbf{k}_B = \text{Boltzmann's constant} = 1.38 \times 10^{-23} \text{ J/K}^o$$

$$\mathbf{k}_B = \mathbf{R} / \mathbf{N}_o \qquad (\mathbf{R} \text{ is gas constant; } \mathbf{N}_o \text{ is Avodagro's number})$$

Photon Gas

Because photons can be created and destroyed, the number of photons in a box is not fixed, but fluctuates about some average value. In describing a gas of photons, we use their **number density** (how many photons per unit volume, on average) and **energy density** (how much energy per unit volume, on average).

The total energy density $\mathbf{U}_\gamma$ of electromagnetic waves in equilibrium is related to the temperature by:

$$\mathbf{U}_\gamma = (8\pi^5 \mathbf{k}_B{}^4 / 15 \mathbf{h}^3 \mathbf{c}^3)\, \mathbf{T}^4, \qquad (3)$$

where $\mathbf{h}$ is Planck's constant (neutrinos obey a similar, but not identical, expression). Substituting, one finds

$$\mathbf{U}_\gamma = 7.565 \times 10^{-16}\, \mathbf{T}^4 \quad (\text{J/m}^3). \qquad (4)$$

where $\mathbf{T}$ must be quoted in ^{o}K, and the resulting $\mathbf{U}_\gamma$ has units of J/m^3.

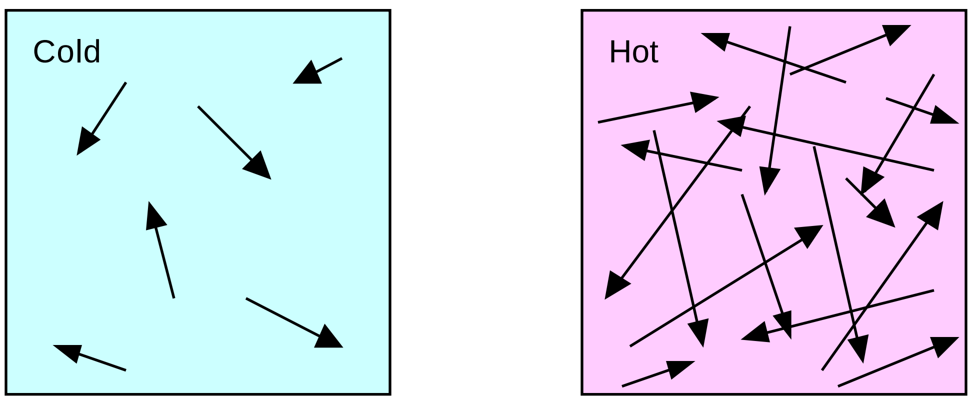
Similarly, the number density found after summing over all photon energies is

$$\mathbf{N}_\gamma = 60.42\ (\mathbf{k}_B\mathbf{T}/\mathbf{hc})^3 \qquad (5)$$

or

$$\mathbf{N}_\gamma = 2.02 \times 10^7\ \mathbf{T}^3\ (\mathrm{m}^{-3}). \qquad (6)$$

where the temperature **T** must be quoted in ^{o}K, and the resulting $\mathbf{N}_\gamma$ has units of m^{-3}. Eq. (6) shows that the photon density increases with temperature:



(a) (b)

Dividing (3) by (5) the average energy per photon is then

$$[\textit{average energy per photon}] = 2.7\ \mathbf{k}_B\mathbf{T}. \qquad (7)$$

Note that for both particles with mass (2) and photons (7), the typical energy scale per particle is $\mathbf{k}_B\mathbf{T}$.

Example

Find the energy density and the number density of photons at **T** = 2.7 ^{o}K.

Solution:

From Eq. (4) $\mathbf{U}_\gamma = 7.565 \times 10^{-16}\ (2.7)^4 = 4.0 \times 10^{-14}\ \mathrm{J/m^3}$.

From Eq. (6) $\mathbf{N}_\gamma = 2.02 \times 10^7\ (2.7)^3 = 4.0 \times 10^8\ \mathrm{m^{-3}}$.

Note: In the example,
•average energy per photon = 4.0 x 10^{-14} / 4.0 x 10^{8} = 10^{-22} J.
•wavelength of typical photon = λ = **hc** / **E** = (6.63 x 10^{-34})(3.0 x 10^{8})/10^{-22} = 0.2 cm (as expected for microwaves).

Microwave Radiation and the Big Bang

We know from the amount of debris hitting the Earth that "empty space" is not empty. Among the elementary particles hitting us from space are
•neutrinos from the Sun: about 6 x 10^{14} per m^2 per second (area facing the Sun)
•cosmic rays: a couple of hundred per m^2 per second.

In 1964, Arno Penzias and Robert W. Wilson found that space is filled with low energy microwaves, using an instrument sensitive to 7.35 cm wavelength. Verification that the microwaves actually have an equilibrium distribution of wavelengths took many more years of observation at different wavelengths. Current observation supports a temperature of 2.73 ± 0.05 ^{o}K. Recent experiments using satellite-based detectors confirm that the 3 ^{o}K microwave radiation is present uniformly in all directions of space; it is not associated with specific stars or the Milky Way.

We put Hubble's law together with the 3 ^{o}K microwave radiation together to form the Big Bang model:
•the Universe is expanding,
  •at one time the Universe must have been more dense
•the microwave radiation is "relic" radiation present since the beginning of time,
  •at one time the Universe must have been hotter, since energy density increases with **T**
•the hot Big Bang took place 7 - 14 billion years ago, according to today's value of **H**.