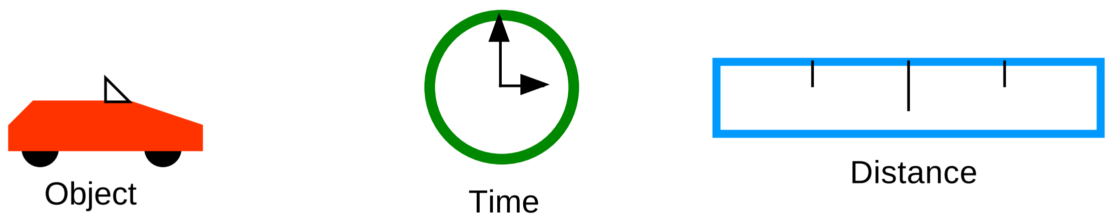
Demonstrations: none
Text: Fishbane 2-1, 2-2, 2-3, 2-4, 2-5
Problems: 15, 26, 27, 45, 48 from Ch. 2

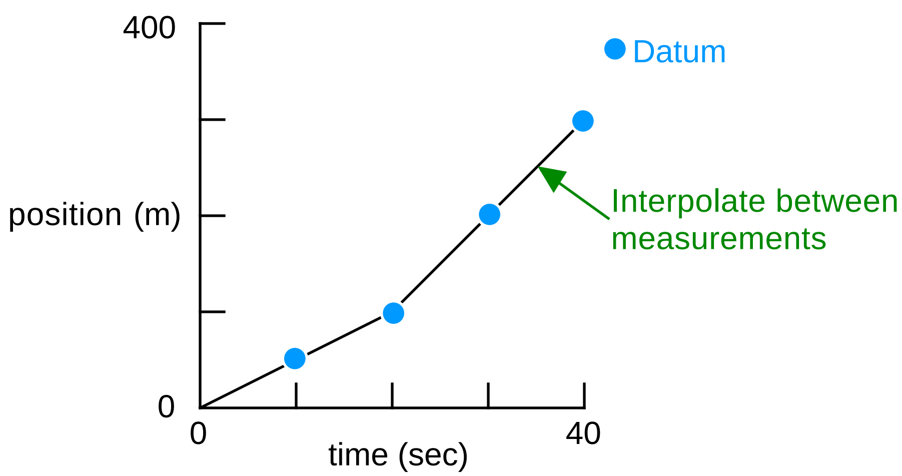
What's important:
•relationships between displacement, velocity, acceleration
•permutations of two basic equations for motion in one dimension

Position and time

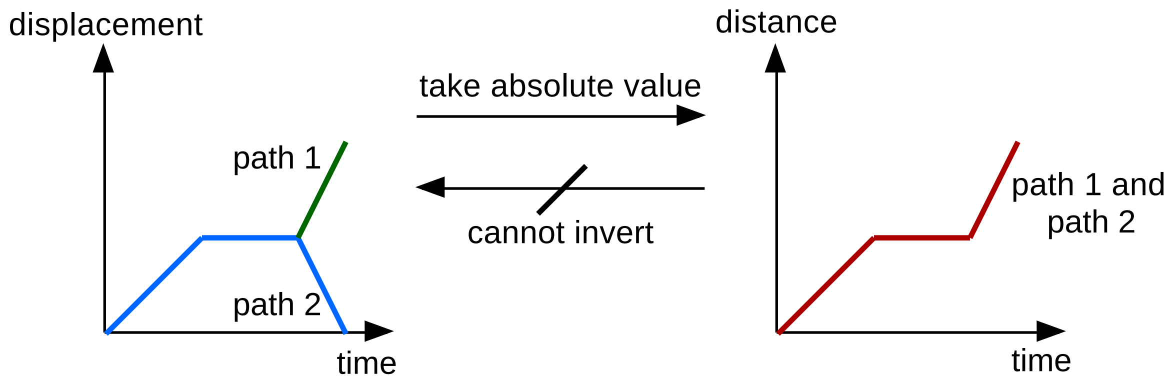
Kinematics deals with movement, the change in position of an object as a function of time. For example, we can use a clock and (very long) meter stick to measure the motion of an object like a car



from which we can construct a position *vs*. time graph:



From the position, or **displacement**, we can construct the **distance** covered by the moving object. Distance is always positive, and depends upon the path followed in the motion. Although distance can be determined by displacement, the reverse is not true: you **cannot** obtain displacement from distance. Consider two different paths, 1 and 2, on the following position *vs*. time graph:



The paths have the same distance *vs*. time graph, which cannot be uniquely inverted to give the original displacement *vs*. time graph.

Rates of change

Displacement and distance change with time. One can use two crude measures of the rate of change of displacement or distance, namely

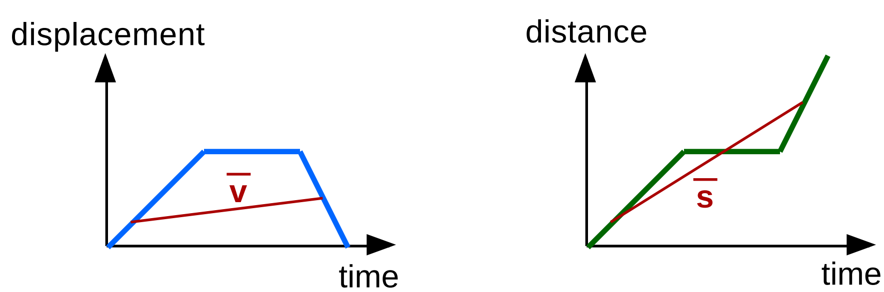
$$\text{average velocity} = \bar{\mathbf{v}} = \frac{\mathbf{x}_2 - \mathbf{x}_1}{\mathbf{t}_2 - \mathbf{t}_1} \qquad \text{(independent of path)}$$

$$\text{average speed} = \bar{\mathbf{s}} = \frac{\text{change in distance}}{\text{change in time}}$$

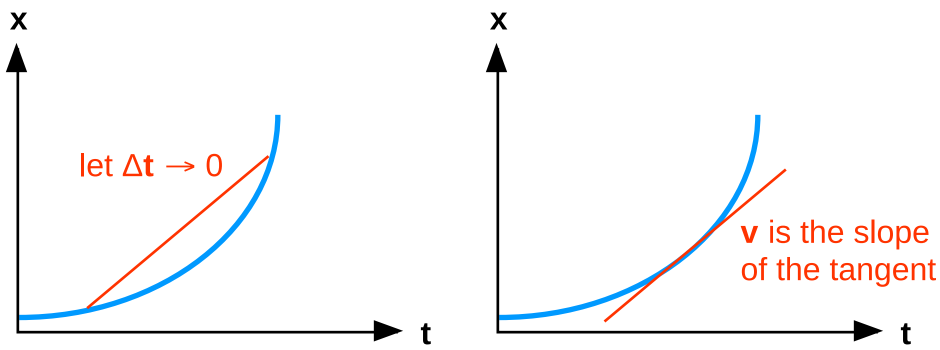
Average speed and average velocity can be obtained from the slope of the line joining the two (position, **t**) or (distance, **t**) points:

$$\text{slope} = \frac{\mathbf{x}_2 - \mathbf{x}_1}{\mathbf{t}_2 - \mathbf{t}_1} = \frac{\Delta \mathbf{x}}{\Delta \mathbf{t}}$$

Clearly, the average **v** and **s** can have quite different values:



We can define an instantaneous velocity **v** (or instantaneous speed **s**) by finding the slope of the tangent to a point on the curve:



Note that **v** has a sign (positive or negative), but **s** does not: the instantaneous speed is just the absolute value of the instantaneous velocity:

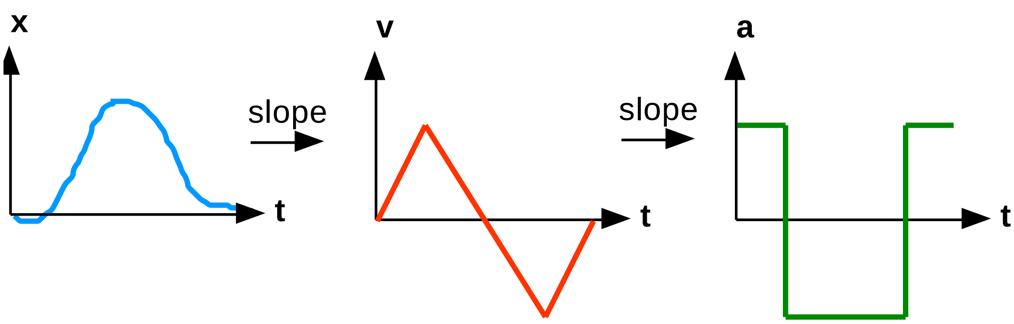
$$\mathbf{s} = |\mathbf{v}|.$$

Acceleration

We can go to higher order rates of change by looking at the rate of change of the velocity (since **s** = |**v**|, then we will deal with velocities rather than speeds).

$$\text{average acceleration} = \bar{\mathbf{a}} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{\mathbf{t}_2 - \mathbf{t}_1} \qquad \text{(independent of path)}$$

$$\text{instantaneous acceleration} = \frac{\Delta \mathbf{v}}{\Delta \mathbf{t}} \qquad \text{as } \Delta \mathbf{t} \to 0$$



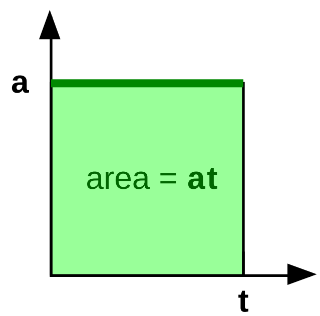
Constant acceleration

We showed above that one takes the slopes of time-evolution graphs to obtain rates of change:

x *vs.* **t** --slope--> **v** *vs.* **t** --slope--> **a** *vs.* **t**

To obtain **x** from **v**, or **v** from **a** (that is, to proceed in the opposite direction from the rates), one takes areas under the curves of time-evolution graphs. For example, a car travelling at a constant speed of 100 km/hr (**s**) covers a distance of 100 km (**st**) in a time of 1 hour (**t**). The distance is the product of **s** and **t**, and is the area under the **s** *vs* **t** graph. We apply this to constant acceleration.

a → **v**



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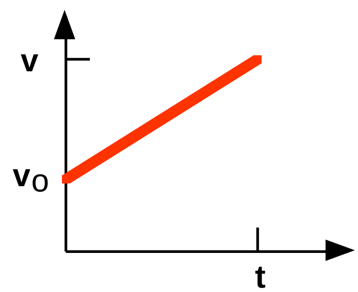


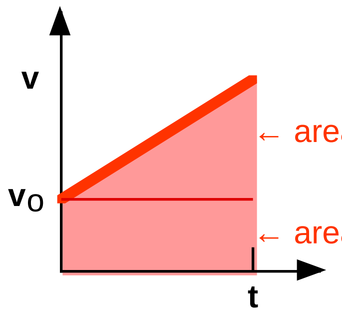
The area under the curve gives the change in **v** (that is, $\Delta\mathbf{v} = \mathbf{v} - \mathbf{v}_o$), NOT **v** itself.

From the graph of **constant acceleration** *vs* **t**,

$$\Delta\mathbf{v} = \text{area under } \mathbf{a} \textit{ vs } \mathbf{t} = \mathbf{at}$$
$$\mathbf{v} - \mathbf{v}_o = \mathbf{at}$$
$$\mathbf{v} = \mathbf{v}_o + \mathbf{at} \qquad (1)$$

Equation (1) shows that the **v** *vs* **t** curve should be a straight line with a y-intercept of $\mathbf{v}_o$.

v → **x**



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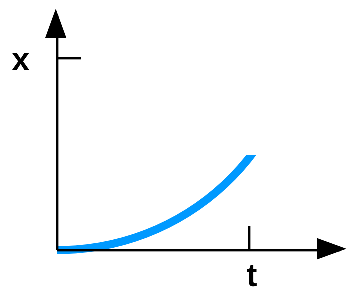

$$\Delta \mathbf{x} = \text{area under } \mathbf{v} \textit{ vs } \mathbf{t} = (\mathbf{v} - \mathbf{v_o})\mathbf{t}/2 + \mathbf{v_o t}$$

$$\mathbf{x} - \mathbf{x_o} = (\mathbf{v} + \mathbf{v_o})\mathbf{t}/2$$

$$\mathbf{x} = (\mathbf{v} + \mathbf{v_o})\mathbf{t}/2 \qquad (\text{if } \mathbf{x_o} = 0) \qquad (2)$$

Although (2) looks like a linear equation in time (whereas the **x** *vs.* **t** is anything but linear), in fact **v** contains time dependence. Substituting (1) into (2) to show the explicit time-dependence gives

$$\mathbf{x} = (\mathbf{v_o} + \mathbf{at} + \mathbf{v_o})\mathbf{t}/2 = (2\mathbf{v_o} + \mathbf{at})\mathbf{t}/2$$

$$\mathbf{x} = \mathbf{v_o t} + (1/2)\mathbf{at}^2 \qquad (3)$$

Equations (1) and (2) can be written in other ways as well. For example, since the plot of **v** *vs.* **t** is linear, then the average velocity $\mathbf{v_{av}}$ is just $(\mathbf{v} + \mathbf{v_o})/2$. Hence, equation (2) also can be written as

$$\mathbf{x} = \mathbf{v_{av} t} \qquad (4)$$

Alternatively, one could invert (1) to find **t**,

$$\mathbf{t} = (\mathbf{v} - \mathbf{v_o})/\mathbf{a}$$

and substitute the result into (2) to obtain

$$\mathbf{x} = (\mathbf{v}^2 - \mathbf{v_o}^2) / 2\mathbf{a} \qquad (5)$$