Demonstrations: none
Text: Fishbane 1-6, 3-1, 3-2, 3-3
Problems: 4, 7, 13, 24, 27 from Ch. 3

What's important:
•vectors and their components
•vector addition, products

Kinematics in Two and Three Dimensions

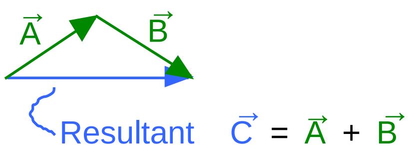
The kinematics of 1-dimensional motion that we discussed in the previous two lectures can be generalized to more than one dimension through the use of vectors.

Scalar: magnitude only, no direction
Vector: magnitude and one direction
Tensor: magnitude and more than one direction.

The length of vector $\vec{A}$ is denoted by $|\vec{A}|$. In Cartesian component notation, vector **A** is represented by (A_x, A_y, A_z). Here, we consider addition, multiplication and functional operations on vectors.

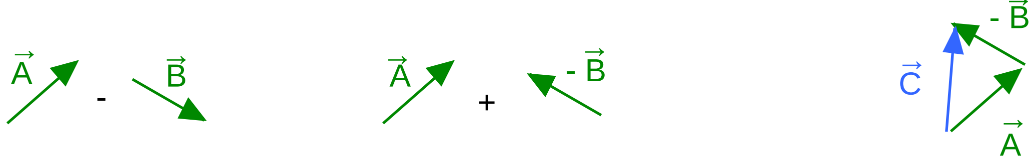
<u>Addition and subtraction:</u>

Addition rule: put tip of $\vec{A}$ to tail of $\vec{B}$, **resultant** runs from tail of $\vec{A}$ to tip of $\vec{B}$.



In components: A + B = C = (A_x+B_x, A_y+B_y, A_z+B_z).

$\vec{A} - \vec{B}$ Form negative of $\vec{B}$, then add as usual.



In components: A + B = C = (A_x-B_x, A_y-B_y, A_z-B_z).

Scaler times vector:

$$a\vec{A} = \vec{A} + \vec{A} + \vec{A} \,.....\qquad \text{“a” times.}$$

In components: $a\mathbf{A} = (aA_x, aA_y, aA_z)$.

Multiplication:

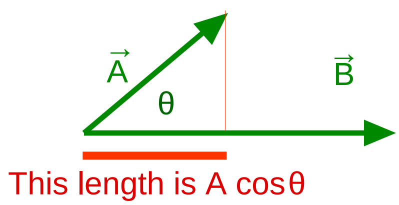
There are three products that one can form from vectors, two of which (the **dot** and **cross** product) are needed in this course. The **dot product** of two vectors is a scalar quantity, and hence the dot product also is called the **scalar product**. The notation for the dot product, and its operation, are:

$$\vec{A} \bullet \vec{B} = |\vec{A}|\,|\vec{B}| \cos\theta$$

here's the dot

this is the angle between the vectors

This operation is equivalent to taking the projection of one vector times the length of the other:



Note that the dot product of a vector with itself is just the square of the vector's length

$$\vec{A} \bullet \vec{A} = |\vec{A}|\,|\vec{A}| \cos 0 = A^2$$

In components:

$$\vec{A} \bullet \vec{B} = A_xB_x + A_yB_y + A_zB_z$$

The other product of vectors needed for this course is the **cross product**. Because the cross product results in a vector, it is also referred to as the **vector product**.

$$\vec{C} = \vec{A} \times \vec{B}$$

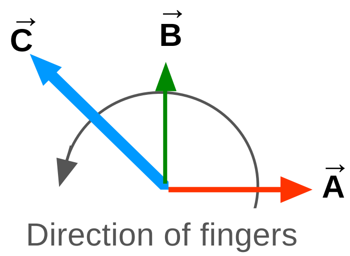
here's the cross

The magnitude of the cross product is

$$|\vec{C}| = |\vec{A}|\,|\vec{B}| \sin\theta$$

this is the angle between the vectors

Finally, the direction of the cross product is given by the right hand rule:
•the new vector is perpendicular to the plane of the vectors **A** and **B** in the product
•the direction of the vector **C** can be found by curling the fingers of your right hand from **A** to **B** (through the smaller angle). Your thumb points in the direction of **C**.



Use fingers of right hand to move from $\vec{\mathbf{A}}$ to $\vec{\mathbf{B}}$. Thumb points along $\vec{\mathbf{C}} = \vec{\mathbf{A}} \times \vec{\mathbf{B}}$.

In components:

$$\vec{A} \times \vec{B} = (a_y b_z - a_z b_y \,,\; a_z b_x - a_x b_z \,,\; a_x b_y - a_y b_x)$$

<u>Derivatives of vectors</u>

Our kinematic quantities **v** and **a** are vectors, just as is the displacement:

$\vec{v}$ = velocity vector $\quad\quad$ $\vec{a}$ = acceleration vector

We can write vector equations for $\vec{v}$, $\vec{a}$:

$$\vec{v} = \frac{d\vec{R}}{dt} \qquad\qquad \vec{a} = \frac{d\vec{v}}{dt}$$

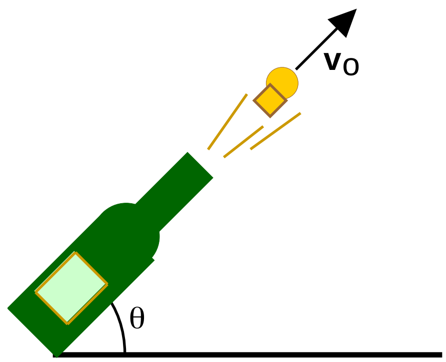
What does the derivative d../d**t** mean in these equations? In Cartesian coordinates

$$\frac{d\vec{R}}{dt} = \left(\frac{dx}{dt} , \frac{dy}{dt} , \frac{dz}{dt} \right)$$

$$= \left(\frac{\Delta x}{\Delta t} , \frac{\Delta y}{\Delta t} , \frac{\Delta z}{\Delta t} \right) \qquad \text{as } \Delta t \to 0$$

Projectile Motion

An age-old problem in kinematics is the motion of projectiles. Consider a bullet from a gun, or, as shown here, a cork from a bottle of champagne:



The initial velocity is $\mathbf{v_o}$. Once the projectile leaves the bottle, it is subject only to forces from gravity and air resistance. We neglect the latter. The initial velocity has components

horizontal (**x**)	$\mathbf{v}_{o,x} = \mathbf{v}_o \cos\theta$
vertical (**x**)	$\mathbf{v}_{o,y} = \mathbf{v}_o \sin\theta$

Only the vertical component is subject to change because of the acceleration due to gravity:

$\mathbf{a}_x = 0$ $\qquad$ $\mathbf{a}_y = -\mathbf{g}$ $\qquad$ ($\mathbf{g} = -9.8$ m/s^2)

The kinematic equations become:

$$\mathbf{x} = \mathbf{v}_{o,x}\,\mathbf{t} = \mathbf{v}_o \cos\theta \;\; \mathbf{t}$$

$$\mathbf{y} = \mathbf{v}_{o,y}\,\mathbf{t} + 1/2\,\mathbf{a}_y\mathbf{t}^2 = \mathbf{v}_o \sin\theta \;\; t - 1/2\,\mathbf{gt}^2.$$