Lecture 18 - Dynamics

What's important:
· Newton's three laws of dynamics
Demonstrations:
· force constant of a spring
· scales for examples
· clear plastic sheet for force on hand
Text: Fishbane 4-1, 4-2, 4-3, 4-5
Problems: 7, 13, 20, 56, 67 from Chap. 4

Dynamics

Kinematics permits us to describe the motion of particles expressed in terms of **x** and its rates of change **v** and **a**. **Dynamics** relates the motion of particles to the forces between them. Together, they constitute **mechanics**. In these lectures, we examine **Newtonian classical** mechanics:
· formulated by Newton (1642-1727) in terms of **x**, **v** and **a**.
· classical --> speeds small compared to the speed of light.

The first law tells us what happens if nothing happens:

Newton's First Law

An object continues in its initial state of rest or motion with uniform velocity unless it is acted upon by an unbalanced force.

Physical intuition: suppose we are in a spaceship moving with respect to the Earth but far away from any planets, etc. Then if the windows of the spaceship were covered over, we could not tell from the inside of the spaceship whether it was moving [this is not true when it is accelerating!]. So, Newton's first law does two things:

· it puts "at rest" or "motion with uniform velocity" on the same footing
· it says that nothing happens to this motion unless the object is acted upon by an unbalanced force.

What does a force do to an object? This is **Newton's Second Law**.

The acceleration a of an object subject to an unbalanced force $\mathbf{F}_{net}$ is directly proportional to $\mathbf{F}_{net}$ and inversely proportional to its mass *m*:

$$\mathbf{a} = \mathbf{F}_{net} / m \qquad \text{or} \qquad \mathbf{F}_{net} = m\,\mathbf{a}.$$

Notes:

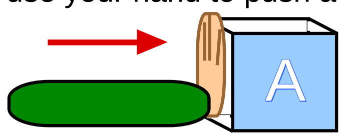
- This is a vector equation and applies component by component:

  $a_x = F_x / m \quad a_y = F_y / m \quad a_z = F_z / m.$
- **a** is parallel to $\mathbf{F}_{net}$.
- $\mathbf{F}_{net}$ is the **vector** sum of all applied forces: $\mathbf{F}_{net} = \Sigma_i \mathbf{f}_i$.

Newton's Third Law deals with the interaction of the body delivering the force and the body receiving the force:

Forces always occur in pairs. If object A exerts a force $\vec{F}$ on object B, then object B exerts a force $-\vec{F}$ on object A. For every action there is an equal and opposite reaction.

Consider an example: use your hand to push a block across a table



We see the motion of the block and think that the only force present is the one acting on the block. But look at the hand: it is slightly flattened up against the block and the reason why it is flattened is the force which the block is exerting on the hand (*demo*).

Some common forces

$1/r^2$ *force* (Newton's law of gravity and Coulomb's law)

$F = Gm_1m_2 /r^2$ gravity

$F = kQ_1Q_2 /r^2$ electrostatics

On the surface of the Earth, $m_1 = m_{Earth}$ $m_2 = m$ (of object) $r = R_{Earth}$

$F = (Gm_{Earth} /R_{Earth}{}^2)\, m$

Using

$G = 6.67 \times 10^{-11}\ \mathrm{Nm^2/kg^2}$ $m_{Earth} = 5.98 \times 10^{24}$ kg $R_{Earth} = 6.37 \times 10^6$ m

we obtain

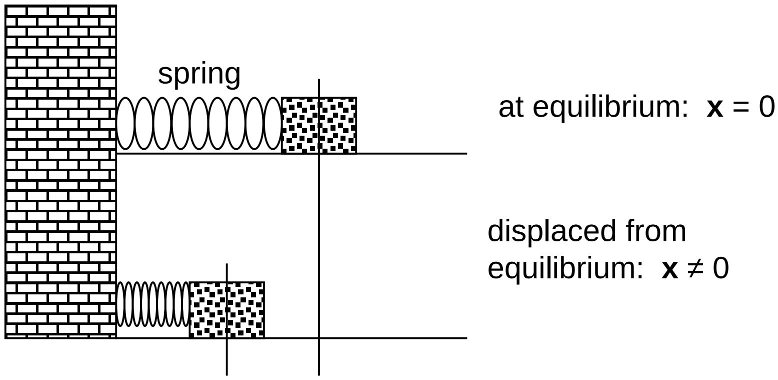
$(Gm_{Earth} /R_{Earth}{}^2) = 9.8\ \mathrm{m/s^2} = g$

Force proportional to distance

The force associated with springs and elastic bands (in fact, most materials) at small deformations is called Hooke's law:

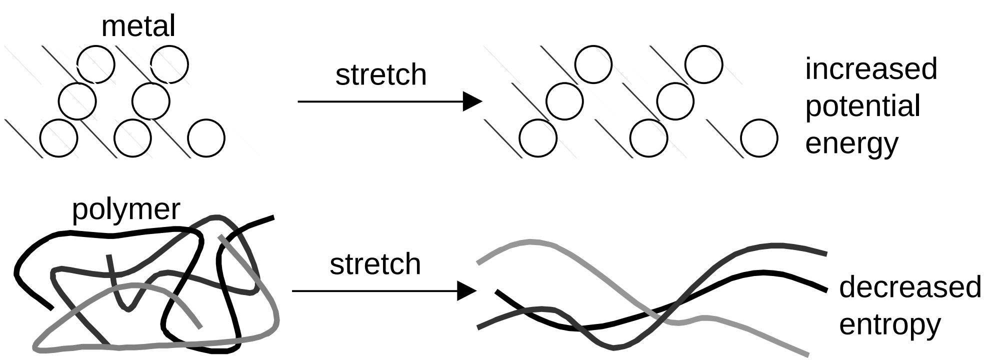
$\mathbf{F} = -k\,\mathbf{x}$,

where **x** is the (vector) displacement from equilibrium.



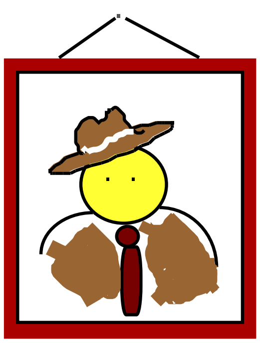
Demo: determine spring constant from plot of *mg* vs *x*.

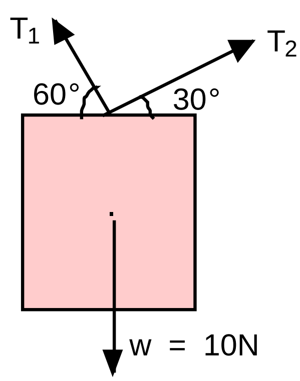
Origin of elasticity



Examples

#1. A picture of weight 10 N is supported by two wires. Find the tension in the wires.





Solution: Since the picture is motionless, then

$$F_{Net} = ma = 0 \quad (2^{nd}\ \text{Law})$$

$$\left.\begin{aligned} F_{Net,\,y} &= T_1 \sin 60° + T_2 \sin 30° - 10 = 0 \\ F_{Net,\,x} &= T_1 \cos 60° - T_2 \cos 30° = 0 \end{aligned}\right\}$$ Two equations, Two unknowns

$T_1 \bullet \frac{\sqrt{3}}{2} = T_2 \bullet (1/2) \qquad T_1 = \sqrt{3}\, T_2$ ($T_1 > T_2$ here, as we can show with Newton scales and one weight.)

Solving $\sqrt{3}\, T_2 \bullet \frac{\sqrt{3}}{2} + T_2 \bullet (1/2) - 10 = 0 \qquad T_2\,(3/2 + 1/2) = 10$

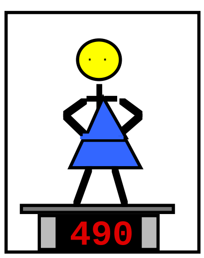
or $T_2 = 5$ N $\qquad T_1 = 5\sqrt{3} = 8.7$ N

#2. A woman of mass 50 kg is standing on a weigh scale in an elevator. What is the reading on the scale when the elevator is

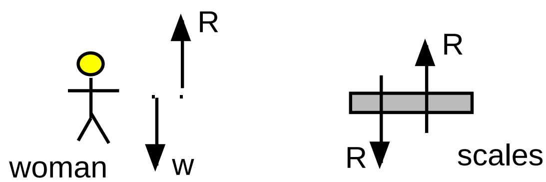
a) not accelerating
b) accelerating upwards at 3 m/s^2
c) accelerating downwards at 3 m/s^2

Solution:

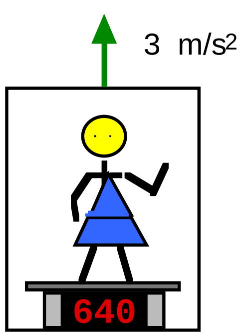
a)



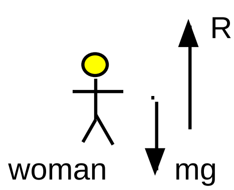
F on top of scales is $\quad mg = 50 * 9.8$
$= 490$ N



b)



If the woman is accelerating upwards, then the reaction force from the scales must not only counteract her weight but must also accelerate her.

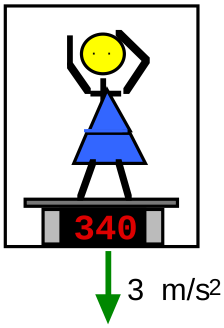


F on top of scales = $mg + ma$

$R = mg + ma$
$= 490 + 150 = 640$ N

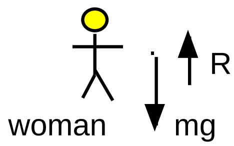
$R - mg$ is the force which accelerates the woman.

c)



If the woman is accelerating downwards, then the reaction force from the scales does not need to fully counteract her weight.

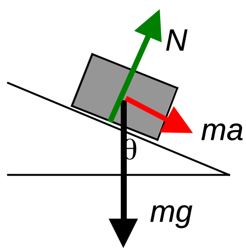
The downward force on the woman is mg - R.



F on top of scales = mg - ma

R = mg - ma
= 490 - 150 = 340 N

#3. Find the forces on a block sliding down a plane.

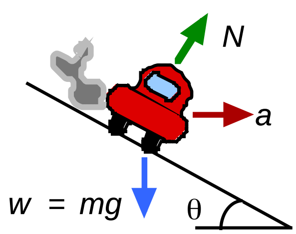


$N = mg \cos\theta$ --> $N \leq mg$

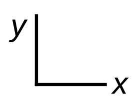
$ma = mg \sin\theta$
--> $a = g\sin\theta$

Check your intuition if you got fouled up on the signs! Here, the normal force does not have to counteract the entire weight of the block, just the component perpendicular to the plane.

#4. Car on a steep embankment, moving in a circle (like a track).



Here, the *x*-component of the normal force *N* provides the centripetal acceleration, so *N* must be larger than *mg* in magnitude.



In the *y*-direction,

$$N \cos\theta = mg$$
$$N \geq mg$$

In the *x*-direction,

$$N \sin\theta = ma$$
$$a = g \tan\theta$$
$$v^2 = Rg \tan\theta$$

NOTE: this is different than Example #3!

#5. Scales in series and parallel