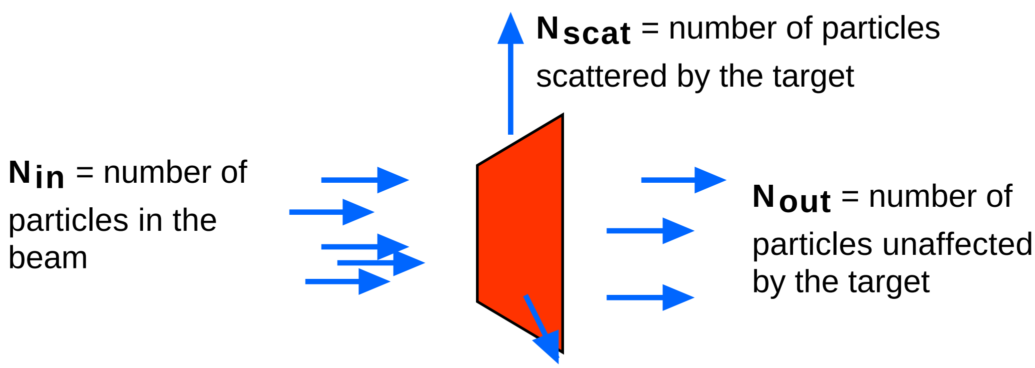
Demonstration: in-plane scattering of beads on a target; sticky marbles; Geiger counter and sources
Text: Mod. Phys. 1.B, 1.C
Problems: 6, 7, 9, 13, 14 from Ch. 1

What's important:
•definition of scattering probability
•relation between probability and cross section
•cross sections and masses for atoms, nuclei and elementary particles

Probabilities and cross sections

Most scattering experiments use a high energy **beam** of particles, which may be electrons, protons, nuclei or many other particle types. To find the probability **P** that a given beam particle will scatter from the target, we compare the number of particles incident upon the target with the number of particles scattered by it:



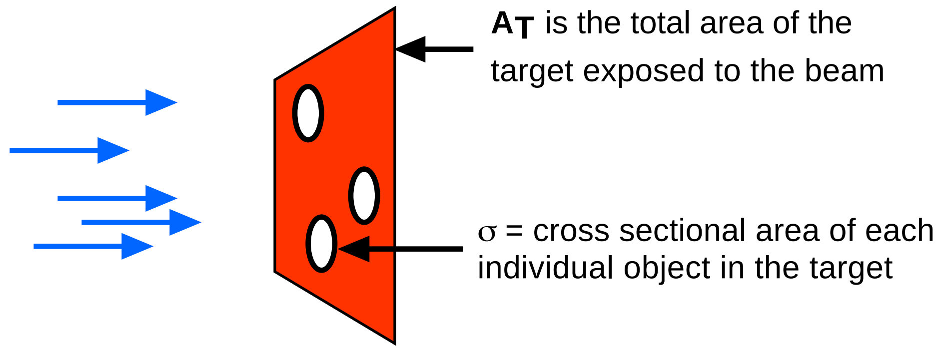
Scattering occurs in the (red) target region

We measure N_{scat} to obtain

$$P = N_{scat} \,/\, N_{in}.$$

Note: we could also find N_{scat} by subtracting N_{out} from N_{in}, but it is very laborious and not very accurate, to count all of the incident and scattered particles.

Having measured **P** in a scattering experiment, we perform an analysis to extract the geometry of the scattering objects. Theoretically, the probability of scattering is equal to the ratio of the effective area of the target objects (as seen by the incident beam of particles) compared to the total area of the target region exposed to the beam:



$$\mathbf{P} = [\text{number of target particles exposed to the beam}]\ \sigma / \mathbf{A_T}.$$

But, the number of target particles exposed to the beam is just the product of $\mathbf{A_T}$ with the number of particles per unit area $\mathbf{n_T}$:

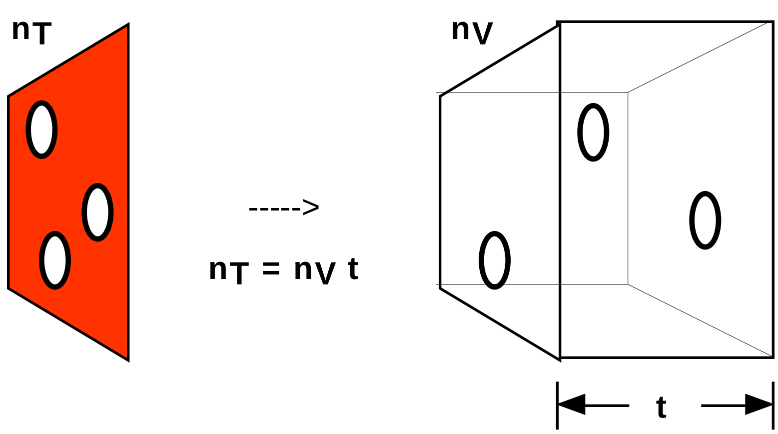
$$[\text{number of target particles exposed to the beam}] = \mathbf{n_T}\ \mathbf{A_T}.$$

Hence,

$$\mathbf{P} = \mathbf{n_T}\ \mathbf{A_T}\ \sigma / \mathbf{A_T} \quad\text{----->}\quad \mathbf{P} = \mathbf{n_T}\ \sigma\ .$$

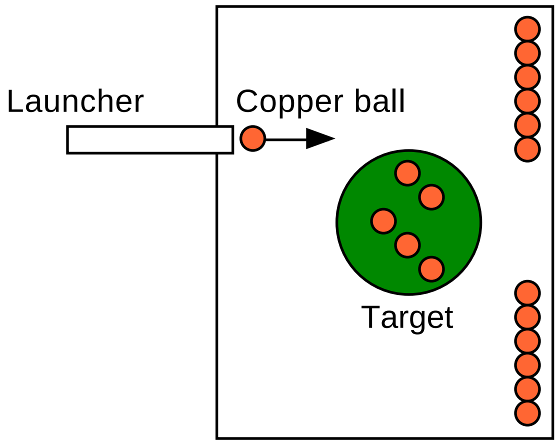
We don’t need to know $\mathbf{A_T}$ (which is hard to measure), just $\mathbf{n_T}$ (which is easy to measure).

We have drawn the target as if it is a two-dimensional plane; in fact, the target is three dimensional. The density of target particles $\mathbf{n_T}$ is a number of objects per unit area. To relate $\mathbf{n_T}$ to the number of objects per unit voume $\mathbf{n_V}$, we use the target thickness $\mathbf{t}$



Demo:

Small copper beads are rolled across a clear plastic sheet into which a hole has been cut. The whole sheet is about 20 cm across, and the hole is about 5 cm in diameter. In the demo, 20 beads are rolled at random positions, and about 5 fall into the hole.



The scattering probability in the demo is

P = 5 / 20 = 0.25.

The number density of the target region is 1 target object in 20 cm, or

$\mathbf{n_T}$ = 1 / 20 = 0.05 cm^{-1} (note that this is the number per unit length!).

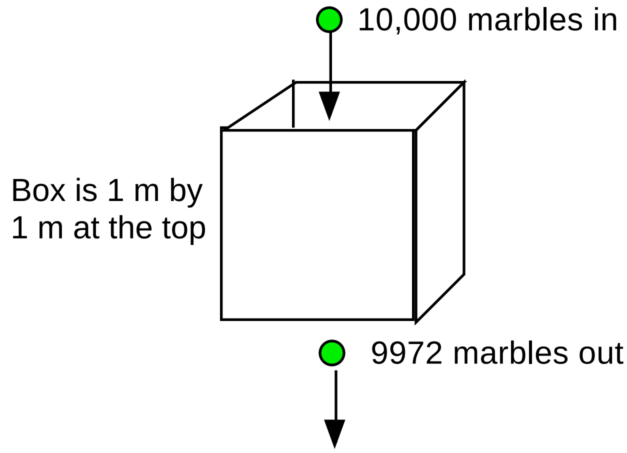
From **P** = $\mathbf{n_T}$ σ, then

0.25 = 0.05 σ, or σ = 5 cm.

(We emphasize that this σ is the one-dimensional analogue of the cross section; here ONLY, n_T is a number per unit length, and σ is a length).

Example (complete version in text)

We want to find the size of an "ideal" spherical apple hidden in a box with no top or bottom. We drop 10,000 sticky marbles at random into the box, covering an area 1 m by 1 m. Of all the marbles dropped in, 28 do not come out the bottom.



$\mathbf{P} = \mathbf{n}_{\mathbf{scat}} / \mathbf{n}_{\mathbf{in}} = 28 / 10^4 = 2.8 \times 10^{-3}$.

$\mathbf{n}_{\mathbf{T}}$ = [number of target objects per unit area] = 1 m^{-2}.

$\mathbf{P} = \mathbf{n}_{\mathbf{T}}\, \sigma \quad \rightarrow \quad \sigma = \mathbf{P} / \mathbf{n}_{\mathbf{T}} = 2.8 \times 10^{-3} / 1 = 2.8 \times 10^{-3}\ m^2$.

This example has the right units, in that $\mathbf{n}_{\mathbf{T}}$ is a number per unit area and σ is an area.

Sizes

What is found experimentally from this scattering technique is:

1. "Low energy" beams of particles measure atomic sizes of 0.1 to 0.2 nm, where 1 nm = 1 nanometer = 10^{-9} m (1Å = 1 Ångstrom = 10^{-10} m).

2. "Medium energy" beams of particles pass into the atom and measure the size of its nucleus - about 2 to 6 fm, where 1 fm = 1 femtometer = 10^{-15} m (a femtometer is also called a Fermi).

nuclear radius / atomic radius ~ 10^{-5}

so

nuclear volume / atomic volume ~ 10^{-15}.

The nuclear radius increases with the number of protons and neutrons **A** in it as

$\mathbf{R} = 1.2\ \mathbf{A}^{1/3}$ fm.

3. "High energy" beams of particles probe inside the nucleus and measure the size of elementary particles.

proton "radius" ~ 1 fm

electron "radius" < 0.1 fm (looks pointlike)

Masses

Different techniques (see text) are used to determine particle masses. One finds:

photon (γ)	$< 5 \times 10^{-65}$ kg
neutrino (ν)	$< 3 \times 10^{-35}$ kg
electron (e^-)	9.11×10^{-31} kg
pion (π)	2.4×10^{-28} kg
proton (p)	1.673×10^{-27} kg
neutron (n)	1.675×10^{-27} kg