Demonstrations:
•block on plane
•balloon with propellor
•conversion of work to P.E.
Text: Fishbane 6-1, 6-2, 6-3
Problems: 14, 15, 17, 27, 41 from Ch. 6

What's important:
•work, kinetic energy, potential energy

Work, Kinetic Energy

In previous lectures, we investigated the effect of a force acting over a period of time. Newton's Second Law is sometimes written in the form

$$F = ma = m\frac{\Delta v}{\Delta t} \qquad \underset{\text{impulse}}{\underline{\underline{F\,\Delta t}}} = m\,\Delta v = \Delta(mv) = \Delta p$$

Force acting
through time
gives impulse

What about a force acting through a distance?

$$F\,\Delta x = \text{Work} = W \qquad \text{(measured in Joules)}$$

Suppose the force produces a constant acceleration **a**. Then

$$W \equiv F\,\Delta x = F\left(\frac{v^2 - v_o^{\,2}}{2\mathbf{a}}\right) = m\mathbf{a}\left(\frac{v^2 - v_o^{\,2}}{2\mathbf{a}}\right) = 1/2\,mv^2 - 1/2\,mv_o^{\,2}$$

At this point, work begins to look like an interesting quantity because it depends only on the end - points (not on the path). Is it true in general? Yes, since for a variable force with no dissipation

$$W = \int_{x_1}^{x_2} F\,dx = \int_{x_1}^{x_2} m\mathbf{a}\,dx = \int_{x_1}^{x_2} m\left(\frac{dv}{dt}\right)dx$$

$$\text{But} \quad \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} \qquad \text{(chain rule)}$$

$$W = \int_{x_1}^{x_2} m \frac{dv}{dx} \bullet \frac{dx}{dt} \bullet dx = \int_{x_1}^{x_2} mv \frac{dv}{dx} \bullet dx$$

$$= \int_{v_1}^{v_2} mv\,dv = \frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2$$

(independant of path)

So we see that the work changes the kinetic energy $1/2\ mv^2$ of the particle. In words,

The work done by an <u>unbalanced</u> force is equal to the change in the kinetic energy of the object.

As a problem-solving technique, construct a free-body diagram to determine the unbalanced force.

In three dimensions, we must generalize $\int F\,dx$. Since force and displacement are both vectors, then we expect

$$W = \int \vec{F} \underset{\substack{\uparrow \\ \text{dot - product}}}{\cdot} \vec{dx} = \int F_x\ dx + \int F_y\ dy + \int F_z\ dz$$

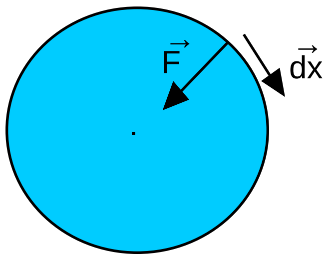
In writing out the dot - product, the direction of dx with respect to F is taken into account by the limits on the integral:

$$\int_{x_a}^{x_b} F\,dx$$

↳ don't fool around with the sign of this when reversing $x_a \leftrightarrow x_b$

Example

Find the work done by the centripetal acceleration for an object executing uniform circular motion.



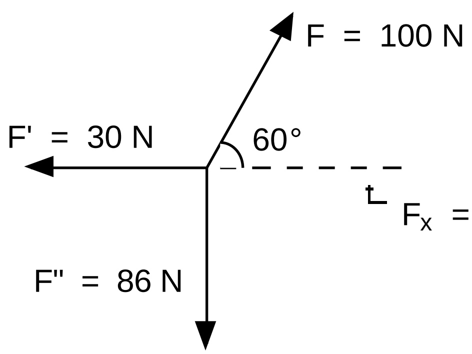
$$\vec{F} = m \frac{v^2}{r^2} \vec{r}$$

$$\vec{F} \qquad \vec{dx} \qquad \int \vec{F} \cdot \vec{dx} = 0$$

It’s no surprise that there is no work done, since there is no change in kinetic energy.

Example

A body moves along the x - axis 20 m subject to the forces F , F' , F'' . What is the change in kinetic energy?



$F_x = 100 \cos 60° = 50$ N

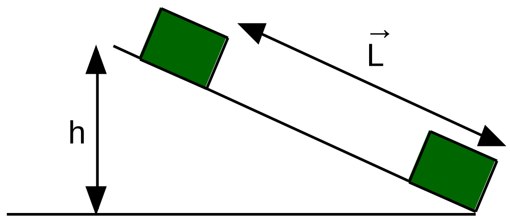
$$W = \int_0^{20} (F_x - |\vec{F'}|)\, dx + \int_0^{0} (F_y - |\vec{F''}|)\, dy + \int_0^{0} 0 \, dz$$

$$= (F_x - |\vec{F'}|) \cdot 20 + (F_y - |\vec{F''}|)\, dy \cdot 0 + 0$$

$$= 20\ (50 - 30) = 400 \text{ N - m} = 400 \text{ J}$$

Potential Energy

Consider the situation of an object sliding down a frictionless incline plane.



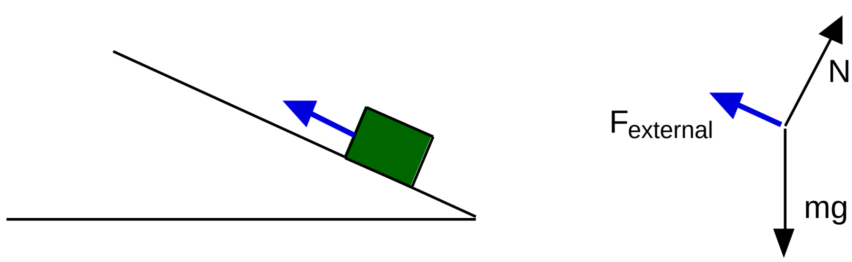
The force due to gravity is mg , and it does work on the block

$$
\begin{aligned}
W &= \vec{F} \cdot \vec{L} \\
&= mg \cdot L \cos\theta \\
&= mgh
\end{aligned}
$$

In turn, the work W results in a change in kinetic energy of the block:

$$mv_{bottom}^2 /2 - mv_{top}^2 /2 = mgh$$

Suppose we now raise the block very, very slowly back up to its original position, by applying a force which just balances the force due to gravity.



The total work is zero, since the forces cancel out in the direction of motion. Similarly, $\Delta K = 0$ since the object is at rest at the top and bottom. The work we have done on the block has not resulted in a change in kinetic energy!

Even though this is a nice consistent picture, we find it unsatisfactory because we are doing work on the object. In order to differentiate what we do as external agents, and what happens to the system in response to our work, we introduce the idea of potential energy. Here

$$W_{\text{done } \textbf{on} \text{ the system}} = \text{increase in potential energy}$$

$$W_{\text{done } \textbf{by} \text{ the system}} = \text{decrease in potential energy}$$

In some sense, the work we have done is "stored" as potential energy, free to be converted into kinetic energy at some later time. The gravitational potential energy is then **U** = mgh. We can now write down a conservation law which states that, in the absence of friction

$$\Delta E = 0 \qquad \text{where} \qquad E = \mathbf{U} + K \qquad \text{(conservation of energy)}$$

$\mathbf{U}$ has replaced W

or

In the absence of dissipative forces [heat, friction...] and of forces for which it is not possible to define a potential energy, the total mechanical energy of a system is constant. [*dissipative forces = friction, viscosity, etc.*]

Example

The work required to separate two bodies attracted by gravity from $\mathbf{r} = R$ to $\mathbf{r} = \infty$, where $\mathbf{r}$ is the distance of separation, is

$$\int_R^\infty F\, d\mathbf{r} = \int_R^\infty G \frac{M_1M_2}{\mathbf{r}^2}\, d\mathbf{r} = G\, M_1M_2 \frac{1}{-2+1}\, \mathbf{r}^{-2+1} \Big|_R^\infty$$

$$= -G\, M_1M_2 \left(\frac{1}{\infty} - \frac{1}{R} \right)$$

$$= \frac{G\, M_1M_2}{R}$$

This work <u>raises</u> the gravitational potential energy of the objects. If we want to say $\mathbf{U}(\mathbf{r} = \infty) = 0$ [i.e. objects at infinite separation have no gravitational PE], then

because PE of objects decreases
as objects come together

$$\mathbf{U}\,(R) \;=\; -\,\frac{G\,M_1 M_2}{R}$$

The **-** sign should not bother us. The “zero” of the potential energy is not defined because we can only evaluate the difference in potential energy arising from the work.

quantity	formula	zero
K	$1/2\ mv^2$	OK. $v = 0$
W	$\int \vec{F} \cdot \vec{dx}$	OK. F or dx = 0 or F dx
U	$\Delta\mathbf{U}$ obtained from W	unknown