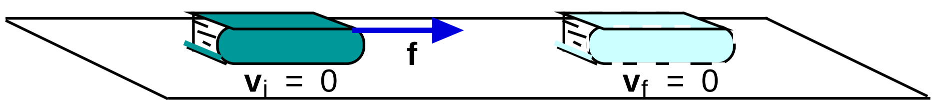
Demonstrations: none
Text: Fishbane 6-4, 6-5, 7-1, 7-3
Problems: 61, 63 from Ch. 6; 11, 22, 33 from Ch. 7

What's important:
•conservation of energy; power

Conservative Forces

Consider what happens when we slide a book across a table against a frictional force



We do work on the book, $W \neq 0$, but $v_i = v_f = 0$ and there is no change in K: $\Delta K = 0$. Further, there is no change in **U**: after we have stopped pushing the book, it does not move back into its original position (*i.e.* the potential energy of the book hasn't changed, so the book can't reduce its potential energy by moving to its original position)! So, friction does not have a potential energy **U** associated with it.

We say that a force like gravity is a **conservative force**: it has a potential energy which depends on position. Friction is a **non-conservative (or dissipative) force** with no potential energy.

Are there other differences between conservative and non-conservative forces?

Gravitation:

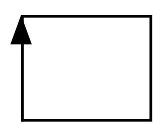
$$W = \int_{x_i}^{x_f} F\,dx = \underline{\underline{\mathbf{U}_f - \mathbf{U}_i}}$$

↳ depends on position of endpoints

Friction:

$$W = \underline{\underline{\int_i^f \mathbf{f}\,dx}}$$

↳ depends on the total path from **i** to **f**.



move book in a closed path,

$$\int \mathbf{f}\,dx \neq 0$$

So, in a conservative force, the work depends only on end-points of the path; in a non-conservative force, the work depends on the path.

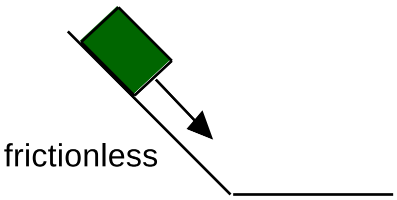
ã 1996 by David Boal, Simon Fraser University. All rights reserved; further resale or copying is strictly prohibited.

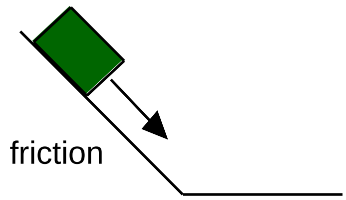
Finally, we can generalize the conservation of energy relation to read:

$$W_{\text{non - cons, by system}} = \Delta E = \Delta(K + \mathbf{U})$$

all the conservative work disappears into potential energy.

Example





$W_{\text{non - cons}} = 0$

$\Delta K = -\Delta \mathbf{U} = W$

work done by gravity lowers the potential energy

$\Delta \mathbf{U} = \mathbf{U}_f - \mathbf{U}_i < 0$

$-\Delta \mathbf{U} > 0$

$W > 0$

$W_{\text{non - cons}} < 0$

$= -\mathbf{f}\,\Delta x$

since opposite directions

$\Delta E = E_{\text{final}} - E_{\text{initial}} < 0$

$E_{\text{final}} < E_{\text{initial}}$

Friction work done by the system has lowered $\mathbf{U}$.

Note: $-\mathbf{f}\,\Delta x$ does not go into block alone; it goes into block and table.

Here, friction has done negative work to lower the total mechanical energy **E** of the system, whereas gravity has done positive work, since $\mathbf{F}_{\text{grav}}$ and displacement are in the same direction.

WARNING: watch out for sign conventions in your work!

Potential and Force

Consider the work done against an attractive force, for which **F** is opposite to **x**. The integral of **F•x** must be negative in going from **X** to **X**+Δ**x**, although the potential energy of the system is increasing since work is being done against an attractive force.

ã 1996 by David Boal, Simon Fraser University. All rights reserved; further resale or copying is strictly prohibited.

$$\int_{X}^{X+\Delta x} F\,dx = -\Delta\mathbf{U} \quad \text{in 1 dimension}$$

Let Δx become sufficiently small that the force is constant over the **X** to **X**+Δ**x** range. Then

$$\int_{X}^{X+\Delta x} F\,d\mathbf{x} \rightarrow F\,\Delta\mathbf{x} = -\Delta\mathbf{U}$$

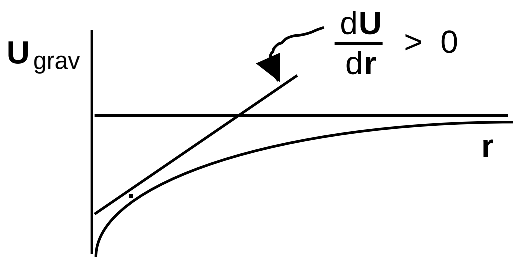
or

$$F = -\frac{\Delta\mathbf{U}}{\Delta\mathbf{x}} \qquad F = -\frac{d\mathbf{U}}{d\mathbf{x}}$$

In other words, the **force is the (negative) derivative of the potential**. Although we have obtained this result for one dimension, it is valid in three dimensions

$$F_x = -\frac{d\mathbf{U}}{d\mathbf{x}} \qquad F_y = -\frac{d\mathbf{U}}{d\mathbf{y}} \qquad F_z = -\frac{d\mathbf{U}}{d\mathbf{z}}$$

What these equations allow us to do is replace a three-dimensional vector **F** by a single scalar function U(x,y,z). Clearly, this only applies for conservative forces. Let's check the minus sign in the equation by an example:



$$F = -\frac{d\mathbf{U}}{d\mathbf{r}} < 0$$

or $\vec{F}$ points towards **r** = 0

$$\text{If} \quad \mathbf{U} = -\frac{G\,M_1M_2}{\mathbf{r}} \qquad \frac{d\mathbf{U}}{d\mathbf{r}} = -G\,M_1M_2\,\frac{d}{d\mathbf{r}}\,\mathbf{r}^{-1} = -(-1)\,G\,M_1M_2\,\frac{1}{\mathbf{r}^2}$$

$$\text{So,} \quad F = -\frac{d\mathbf{U}}{d\mathbf{r}} = -\frac{G\,M_1M_2}{\mathbf{r}^2}$$

towards negative **r.**

ã 1996 by David Boal, Simon Fraser University. All rights reserved; further resale or copying is strictly prohibited.

Power

Power is the **rate of change of energy**:

$$P = \frac{dE}{dt}$$

(measured in Joules / sec ≡ watts.
Electrical utilities like to quote
energy = power x time.
A powerbill uses KW - hr = 1000 W x 3600 sec
= 3.6 x 10^6 J)

(1 hp = 1 horsepower = 746 W)

If we are considering the power delivered by a system doing work, then

$$P = \frac{dW}{dt} = \frac{d}{dt}\int \vec{F}\cdot d\vec{x} = \frac{d}{dt}\int \vec{F}\cdot \frac{d\vec{x}}{dt}\,dt$$

This yields $P = \vec{F}\cdot\frac{d\vec{x}}{dt}$ or $P = \vec{F}\cdot\vec{\mathbf{v}}$

Example

A small motor lifts a load weighing 800 N (~ 90 kg) to a height of 10 m during 20 s. What is the power of the motor?

Solution

The force that the motor must balance is 800 N, and the difference in potential energy in lifting the load is Fh = 800 x 10 = 8,000 J. If the motor does this work in 20 s, its power is

P = 8,000 / 20 = 400 watts.

ã 1996 by David Boal, Simon Fraser University. All rights reserved; further resale or copying is strictly prohibited.