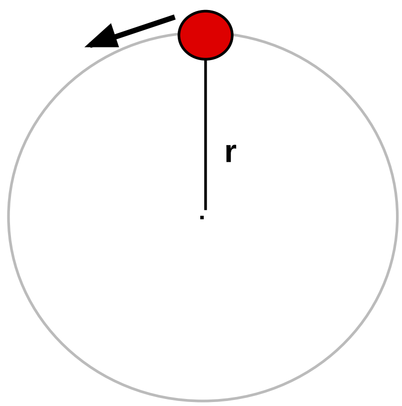
Demonstrations:
•ball on a string, wheel
Text: Fishbane 9-2
Problems: 10, 13, 57 from Ch. 9

What's important:
•definitions of angular variables
•angular kinematics

Rotational Kinematics

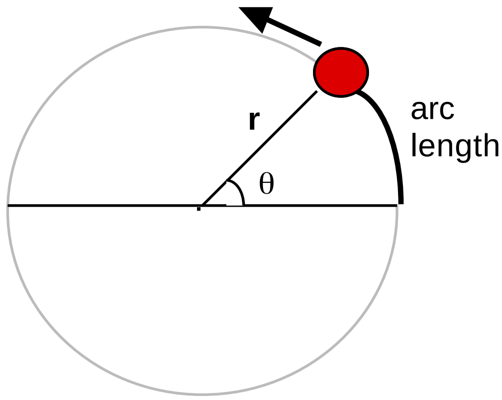
Consider the motion of an object at the end of a string as it moves in a circular path at constant speed,



The speed |**v**| and radius *r* are constant, but the position vector **r** and velocity **v** change continuously, having the form of trigonometric functions. Polar coordinates provide a simpler representation of this situation than Cartesian coordinates do, because only one polar coordinate changes with time, namely the angle θ. We now develop a description of angular motion based upon polar coordinates, starting with uniform circular motion.

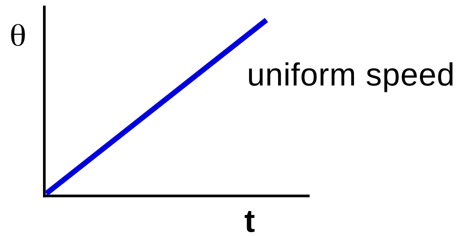
Uniform Circular Motion

In two dimensions, the position of an object can be described using the polar coordinates *r* and θ. If the object is moving in uniform circular motion, then *r* is constant, and the time dependence of the motion is contained in θ. As a kinematic variable, θ is the angular analogue of position.



$$\theta = \frac{\text{arc length}}{\text{radius}} \quad \text{(in radians)}$$

The sign convention is that θ increases in a **counter-clockwise** direction. As a function of time, θ looks like (for uniform circular motion):

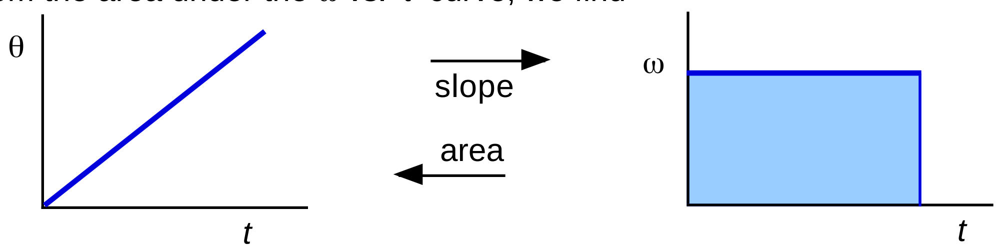


Now, if θ is the angular analogue of position, then the slope of the θ vs. *t* graph is the angular analogue of velocity. We define

angular speed or angular frequency of rotation $\equiv \omega$ (omega) $= \frac{d\theta}{dt}$ $\left(\frac{\text{change in } \theta}{\text{change in time}}\right)$

$\left[\text{or } \frac{\Delta\theta}{\Delta t} \text{ as } \Delta t \to 0 \right]$

We can go back and forth from θ to ω by slopes and areas, just like with **x** and **v**. For example, from the area under the ω vs. *t* curve, we find



$$\theta = \theta_0 + \omega t \quad \text{(at constant } \omega)$$

Two other quantities used to describe uniform circular motion are the period **T** and the frequency *f*. During the period *T*, the object sweeps through one complete revolution, or 2π radians. Hence:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$= \frac{2\pi}{T} \qquad \text{using } \Delta\theta = 2\pi \text{ radians during the period } T$$

The frequency *f* is T^{-1}, so we also have

$$\omega = 2\pi f$$

Note: $\omega = 1$ 1 radian / second

$f = 1$ 1 revolution / second

Angular and linear links

We have a relation between *x* and θ already, namely

[*arc length*] = $r\theta$.

To obtain a relation between **v** and ω, we return to the definition of speed:

speed = d[*arc length*] / d*t* = $d(r\theta) / dt = r\, d\theta / dt = r\omega$.

$$\boxed{v = \omega r} \quad \text{links linear and angular speeds}$$

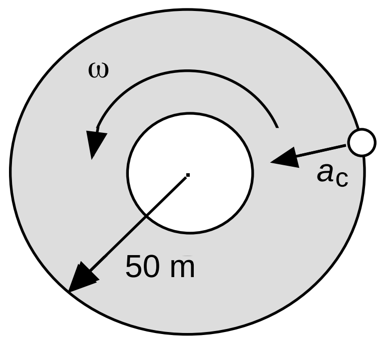
Another linking relation is through the centripetal acceleration $\mathbf{a}_c$. From previous work, there is a centripetal acceleration even at constant speed:

$$a_c = \frac{v^2}{r} = \frac{(\omega r)^2}{r}$$

$$\boxed{a_c = \omega^2 r} \qquad (\text{or } a_c = \omega v)$$

Example

We have a doughnut-shaped space station on which we wish to produce artificial gravity through rotation. What must ω be so that $a_c = g$ at 50 m from the centre of the station?



Using $a_c = \omega^2 r$, we find

$\omega^2 = 9.8 / 50$ --> $\omega = \sqrt{(9.8/50)} = 0.443$ rad/sec.

Note that the units are OK.

Other quantities which we can calculate in this example are:

$$T = \frac{2\pi}{\omega} = \frac{2 \bullet 3.142}{0.443} = 14.2 \text{ sec}$$

which is moderately fast and would not encourage the occupants of the space station to look out the windows. The corresponding speed at the perimeter of the station is:

$$v = \omega r \qquad v = 0.44_3 \bullet 50 = 22.2 \text{ m/s}$$
$$= 80 \text{ km/hr}$$