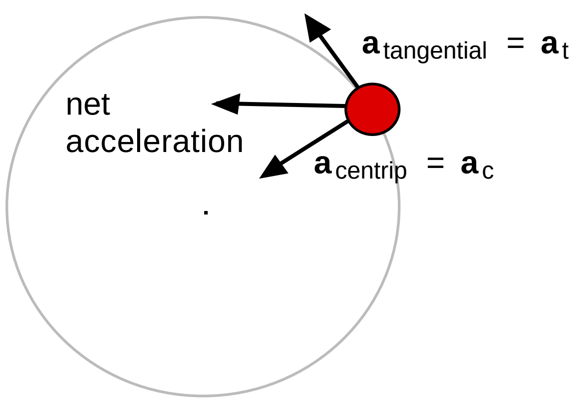
Demonstrations:
•bike wheel, rotating stool
Text: Fishbane 9-2
Problems: 7, 8 from Ch. 9

What's important:
•angular acceleration
•angular velocity and acceleration as vectors

Circular Motion with Variable Speed

Continuing with circular motion, let us consider the case where ω is not a constant. This means that there are two components to the acceleration, a centripetal component towards the centre and a tangential component along the edge:



Just as we define linear acceleration in terms of a change in velocity, we can define an angular acceleration α by a change in angular velocity

$$\text{angular acceleration} \equiv \alpha = \frac{\text{change in } \omega}{\text{change in time}} = \frac{\Delta\omega}{\Delta t}$$

Since $v = \omega r$

$$\alpha = \frac{\Delta\left(\frac{v}{r}\right)}{\Delta t}$$

$$= \frac{1}{r} \frac{\Delta v}{\Delta t}$$ → rate of change of speed. must be tangential acceleration a_t

$$\alpha = \frac{1}{r} a_t$$

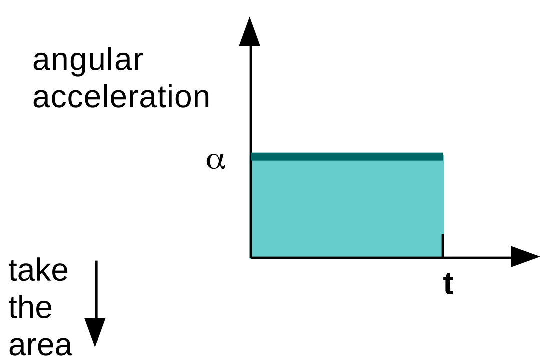
or $a_t = \alpha r$

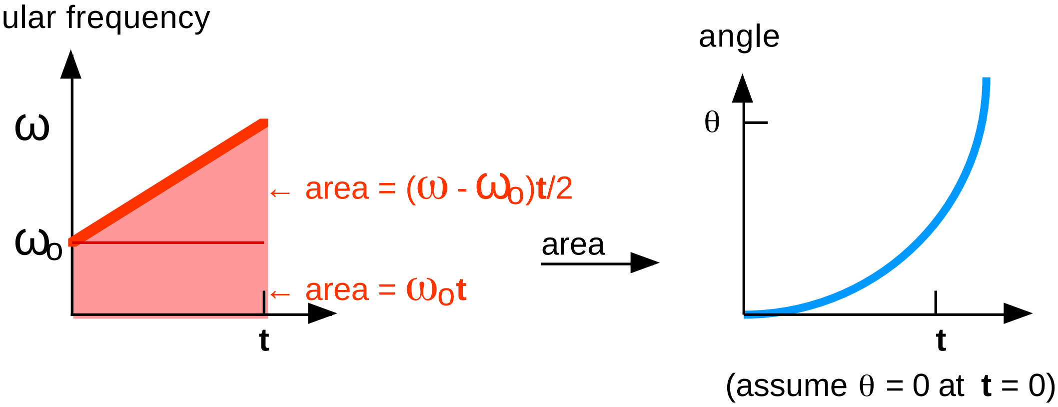
Thus, we have the parallel equations (for constant radius)

arc length $= \theta r$
speed $= |\mathbf{v}| = \omega r$
tangential acceleration $= |\mathbf{a}_t| = \alpha r$

Thus, we would expect to find relations amongst θ , ω , α and t just as with d , v , a and t .

Suppose we have uniform angular acceleration





The area of the angular acceleration graph gives a linear time dependence for the angular velocity:

$$\omega_f = \omega_o + \alpha t \qquad \text{-->} \qquad \alpha t = \omega_f - \omega_o$$

The area of the angular velocity graph gives a quadratic time dependence for the angle:

$$\theta = \omega_0 \mathbf{t} + \frac{1}{2}(\omega - \omega_0)\,\mathbf{t}$$

$$= \omega_0 \mathbf{t} + \frac{1}{2}\alpha \mathbf{t}^2$$

Finally, an alternate expression for the angle which does not explicitly show the time dependence can be found by rearranging the above expressions:

$$\theta = \frac{\omega_f^2 - \omega_0^2}{2\alpha}$$

Example

A carousel, starting from rest, attains a frequency of one revolution every 5 sec after 10 sec. Assuming uniform acceleration, what is the angular acceleration? If the carousel is 10 m in diameter, what is $\mathbf{a}_t$ after 10 sec?

After 10 sec, $\mathbf{f} = \frac{1}{5}$ sec $^{-1}$ $\left(\mathbf{f} = \frac{1}{T}\right)$

$$\omega = 2\pi \mathbf{f} = \frac{2\pi}{5} \text{ rad / sec}$$

Solution

We can use $\omega = \omega_0 + \alpha \mathbf{t}$, with

$$\omega_0 = 0 \quad \text{and} \quad \omega = \frac{2\pi}{5} \text{ at } \mathbf{t} = 10 \text{ sec}$$

[1 rev every 5 seconds -> **f** $= 1/5\ s^{-1}$]

to give the angular acceleration

$$\alpha = \frac{2\pi}{5} \bullet \frac{1}{10} = \frac{\pi}{25} \text{ radians / sec}^2$$

The tangential acceleration, $\mathbf{a}_t$, which is time-independent (if α is constant), can be found from the angular acceleration:

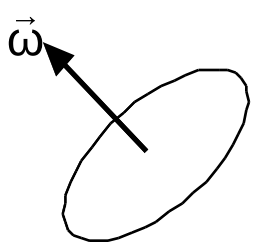
$$\mathbf{a}_t = \alpha \mathbf{r} = \frac{\pi}{25} \bullet 5 = \frac{\pi}{5} = 0.63 \text{ m / s}^2$$

NOTE: even though $\mathbf{a}_t$ is constant, $\mathbf{a}_{tot}$ is not constant, since $\mathbf{a}_c$ changes with **v** as the carousel accelerates.

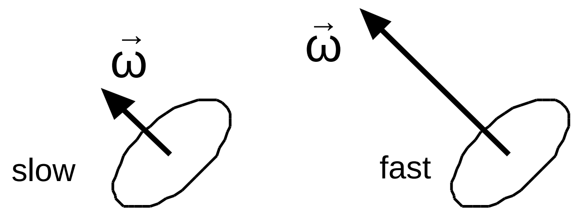
Vectors

Do angular analogues of $\vec{\mathbf{x}}$, $\vec{\mathbf{v}}$, $\vec{\mathbf{a}}$ exist for θ, ω, α ? I.e., are ω, α actually vectors? The answer is yes, as can be seen from the demonstrations.

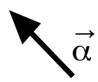
Demos: use bike wheel, with prof seated on rotating stool, to show that $\vec{\omega}$ has a direction



For example, if the angular velocity of the wheel increases, then the length of the ω vector must increase



The increase in ω results from an angular acceleration α. The direction of the change in ω must be the same as the direction of the angular acceleration α:



An angular acceleration α can change the direction of ω as well, with the change in direction of ω pointing in the same direction as α:

If $\vec{\omega}$ turns, but does not change in magnitude, then $\vec{\alpha}$ is not zero.