Demonstrations:
•Wilberforce pendulum
•objects with differing moments of inertia rolling down an incline
Text: Fishbane 9-3, 9-7
Problems: 20, 21, 55, 56 from Ch. 9

What's important:
•work, energy and power in angular variables

Work, Energy and Power

Kinetic Energy

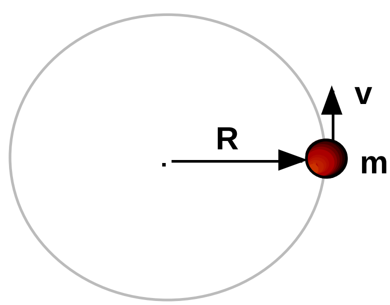
One last group of definitions completes our description of kinematics in angular variables: the angular equivalants of energy, work, *etc.* Let's start off with the kinetic energy **K** for uniform circular motion:

$$\mathbf{K} = \frac{1}{2}\,\mathbf{mv}^2$$

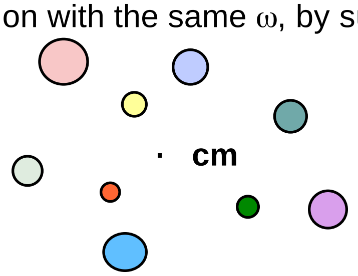
$$= \frac{1}{2}\,\mathbf{m}(\omega\mathbf{R})^2$$

$$= \frac{1}{2}\,(\mathbf{mR}^2)\,\omega^2$$

$$= \frac{1}{2}\,I\,\omega^2$$



We can determine the kinetic energy of a large number of objects, all rotating about the cm position with the same ω, by summing over the individual energies:



each object has the same ω

$$\mathbf{K} = \sum \left(\frac{1}{2}\,I_i\,\omega^2\right)$$

$$= \frac{1}{2}\left(\sum I_i\right)\omega^2$$

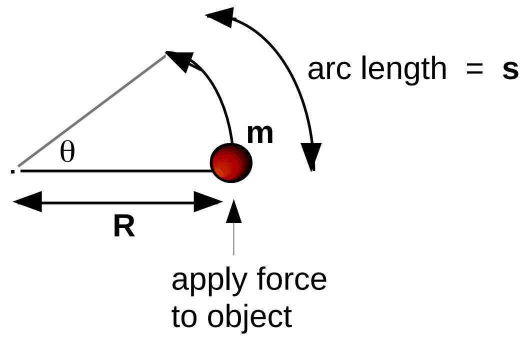
$$= \frac{1}{2}\,I_{TOT}\,\omega^2$$

Note that each moment of inertia $\mathbf{I_i}$ is with respect to the same cm axis. Since each object has the same angular velocity, then the total kinetic energy is proportional to the total moment of inertia.

Work

Work has the same definition as it has in Cartesian coordinates: it is the product of the applied force **F** with the arc length **s** through which the force acts.

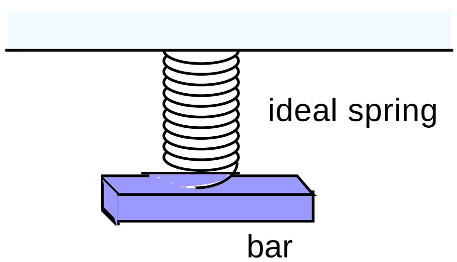
$$
\begin{aligned}
\mathbf{W} &= \mathbf{F}\ \mathbf{s} \\
&= \mathbf{F}\ \mathbf{R}\theta \\
&= (\mathbf{F}\ \mathbf{R})\ \theta \\
&= \tau_z\ \theta
\end{aligned}
$$



where τ_z is the component of the torque τ along **z** - axis

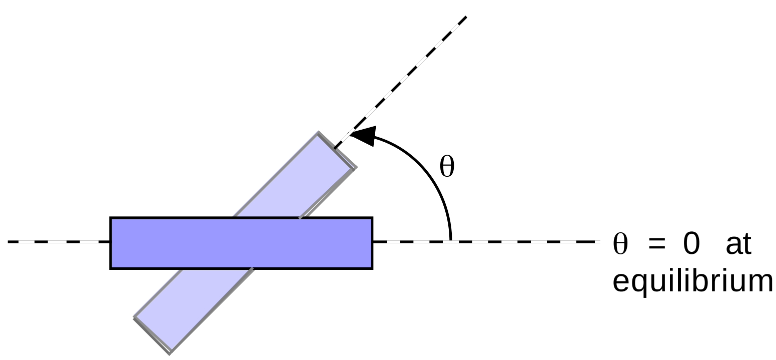
Potential Energy

Work can have both dissipative and non-dissipative (conservative) forms. The conservative forms lead to a potential energy. Consider the torsion spring (demo):



Displace the bar from its equilibrium position by rotating it around the axis of the spring

Looking down through the spring towards the bar:



Restoring torque is $\tau = -k\theta$ (like $F = -kx$, although the k's have different units)

The work is then
$$W = \int \tau \, d\theta$$
$$= \int_0^{\theta_{max}} k\theta \, d\theta$$
$$= \frac{1}{2} k\theta_{max}^2 \quad \text{or just} \quad W = \frac{1}{2} k\theta^2$$

Finally, the potential energy is $U = \frac{1}{2} k\theta^2$

Power

The definition of power in angular variables proceeds as before:

$$\mathbf{P} = \vec{\mathbf{F}} \cdot \vec{\mathbf{V}} = (\mathrm{R}\mathbf{F})\left(\frac{\mathbf{V}}{\mathrm{R}}\right) = \tau_z \cdot \omega$$

(or, since the work $\mathbf{W} = \tau_z \, \theta$ $\quad \mathbf{P} = \frac{d\mathbf{W}}{d\mathbf{t}} = \tau_z \frac{d\theta}{d\mathbf{t}} = \tau_z \, \omega$)

Demo:

Consider two cans with the same mass m that roll down an incline plane after being released from rest at the same height h. One can is filled with a thin liquid soup, while the other is filled with thick stew. The decrease in the potential energy is the same for the two cans: $-mgh$. Hence, the total kinetic energy of each can at the bottom of the incline must be the same. But the division of the total kinetic energy into translational and rotational motion is different because of the different moments of inertia. That is
$$K = I\omega^2 / 2 + mv^2 / 2 = I\omega^2 / 2 + m(\omega r)^2 / 2$$
or
$$K = (I + mr^2)\,\omega^2 / 2.$$
Equating the kinetic energy with the change in potential energy yields

$$mgh = (I + mr^2)\,\omega^2 / 2 \quad \text{-->} \quad \omega^2 = 2gh \,/\, (r^2 + I/m).$$

Demo: soup vs stew

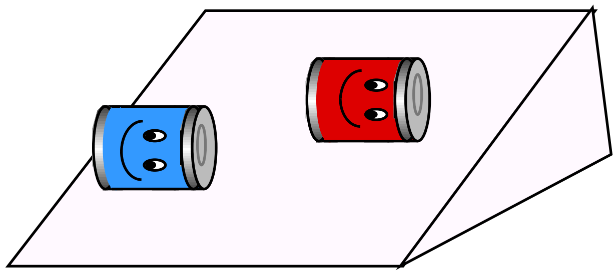
Since the soup doesn't rotate with can,

$$I_{soup} < I_{stew}$$

and hence, for the same K,

$$\omega \text{ of soup} > \omega \text{ of stew.}$$

Soup reaches the bottom of the incline faster than the stew.



Demo: cylinders, spheres and rings

A. Compare two solid spheres: steel and billiard ball (hydrocarbon)
same radius, different masses -> same times
conclude: ω is independent of mass

B. Compare ring and solid cylinder
same radius, different masses -> different times
conclude from A + B: ω depends on mass distribution (I/m)

C. Compare two solid cylinders with different radii
same times
confirms: ω depends on mass distribution (I/m).

$I_{\text{cyl shell}} = mR^2$

$I_{\text{solid cyl}} = (1/2)mR^2$

$I_{\text{solid sphere}} = (2/5)mR^2$