Demonstrations: none
Text: Fishbane 11-1, 11-2, 11-3
Problems: 11, 15, 22, 28, 41 from Ch. 11

What's important:
•conditions for no rotation and no translation

Statics

In linear kinematics, we said that for an object to be stationary, its acceleration **a** must vanish. Applying Newton's Second Law:

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

we find that the condition for no translation of the object is

$$\sum_i \vec{\mathbf{F}}_i = 0$$

for static equilibrium
(up to 3 independent equations).

Newton's Second Law for angular motion is

$$\vec{\tau} = I\vec{\alpha}$$

so that the condition for no angular rotation of the object is

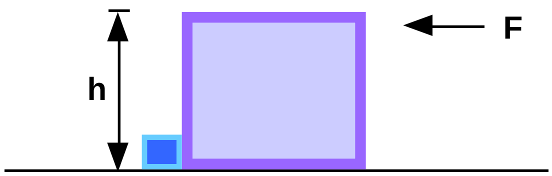
$$\sum_i \vec{\tau}_i = 0$$

for static equilibrium
(up to 3 equations)

Note that if the object is not rotating, then the net torque around every axis that you choose is zero.

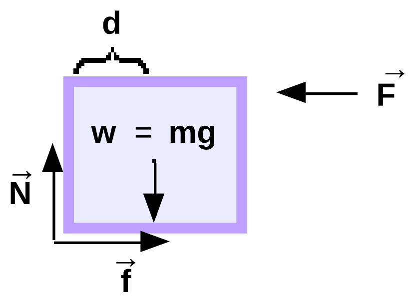
Example 1

A block is pushed against a fixed point **A:**



Find the maximum force before the block rotates.

First, construct a free-body diagram. In addition to the applied force **F**, there is a normal force **N** opposing the weight **mg**, and there is a reaction force **f** opposing the applied force **F**:



Note, $\vec{\mathbf{N}}$ and $\vec{\mathbf{f}}$ act at point **A** when block is about to rotate over stopper.

We can use linear conditions $$\sum_i \vec{\mathbf{f}}_i = 0$$

to provide two conditions on $\vec{\mathbf{N}}$ and $\vec{\mathbf{f}}$.

x - direction: $\vec{\mathbf{f}} = -\vec{\mathbf{F}}$

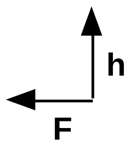
z - direction: $\vec{\mathbf{N}} = -\mathbf{m}\vec{\mathbf{g}} = -\vec{\mathbf{w}}$

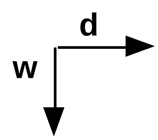
We impose the constraint of no rotational motion by saying that there should be no net torque:

$$\sum_i \vec{\tau}_i = 0$$

$$\underline{\underline{\mathbf{hF}}} \quad - \quad \underline{\underline{\mathbf{dw}}} \quad = \quad 0$$

out of plane — into plane

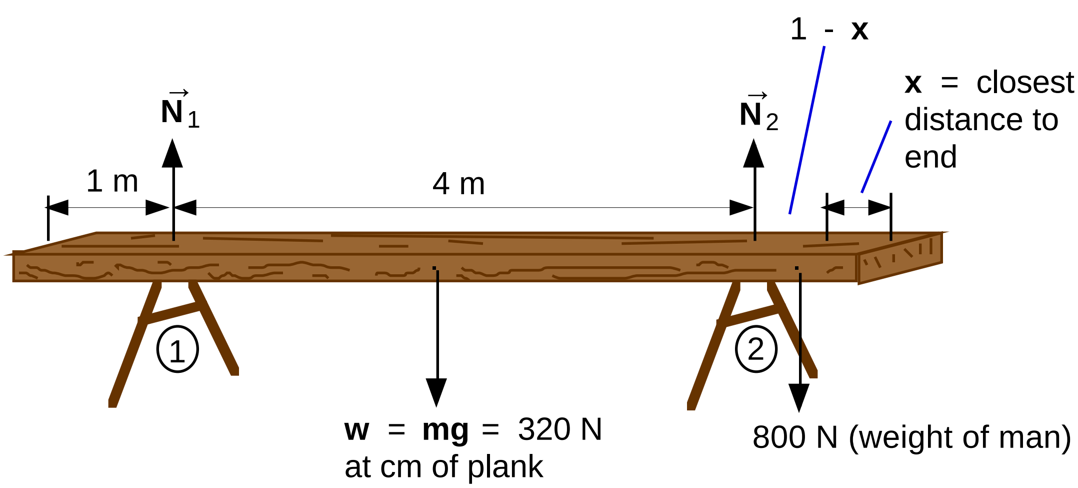




$$\mathbf{F = wd / h}$$

<u>Example 2</u>

A uniform plank of length 6 m and weight 320 N is supported on two trestles, each 1 m from an end of the plank. How close to the end can a man of weight 800 N walk before the plank tips?



Forces in **z** - direction for static equilibrium

$$\mathbf{N}_1 + \mathbf{N}_2 = 320 + 800 = 1120$$

But when the plank is just about to rotate, $\mathbf{N}_1 = 0$

$$\mathbf{N}_2 = 1120 \text{ N at rotation.}$$

Torque equation can be applied at any axis. Usually one looks for the axis of rotation, or finds a convenient axis where many τ 's vanish.

Consider the torque about position (2) :

$$\tau_{net} = -\underbrace{\mathbf{N}_1}_{=\,0}(4) + 320\,(2) + \underbrace{\mathbf{N}_2}_{=\,0}(0) - (1 - \mathbf{x})\,800$$

$$= 640 - 800 + 800\mathbf{x} = -160 + 800\mathbf{x}$$

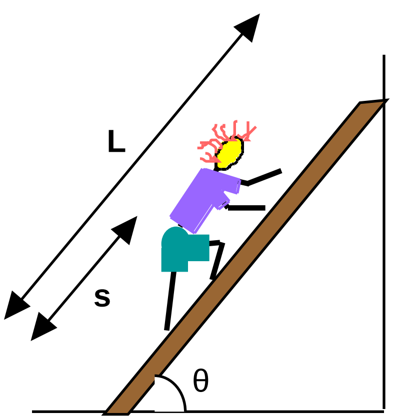
When the torque vanishes

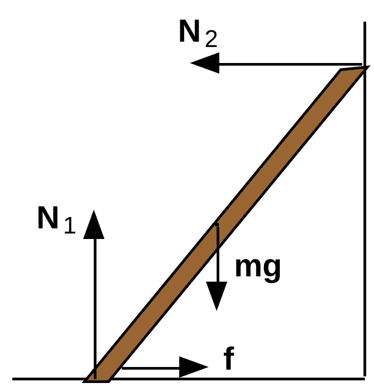
$$-160 + 800\mathbf{x} = 0$$

or $$\mathbf{x} = \frac{160}{800} = 0.2 \text{ m}$$

<u>Example 3</u>

A man is climbing a massless ladder leaning against a frictionless wall. If the coefficient of friction between the ladder and the floor is μ_s , how far up the ladder can the man climb before the ladder slips?





Forces in **x** - direction: $\mathbf{f} = \mathbf{N}_2$ (magnitude) ①

Forces in **z** - direction: $\mathbf{N}_1 = \mathbf{mg}$ (magnitude) ②

Friction law: $\mathbf{f} = \mu \mathbf{N}_1$ ③

Rotation condition: $\mathbf{mg\, s} \cos\theta = \mathbf{N}_2 \mathbf{L} \sin\theta$ ④

① + ② $\mu \mathbf{N}_1 = \mathbf{N}_2$ ⑤ ① + ② + ③ $\mathbf{N}_2 = \mu\, \mathbf{mg}$

② + ⑤ $\mu\, (\mathbf{mg}) = \mathbf{N}_2$ ⑥

④ + ⑥ $\mathbf{mg\, s} \cos\theta = \mu\, \mathbf{mg\, L} \sin\theta$

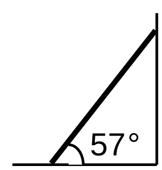
or $\mathbf{s} = \mu \mathbf{L} \dfrac{\sin\theta}{\cos\theta}$

or $\dfrac{\mathbf{s}}{\mathbf{L}} = \mu \tan\theta$ [Independent of **m**!]

Hence, for the person to climb the ladder all the way to **s = L**,

$\tan\theta \geq \dfrac{1}{\mu}$

If $\mu = 0.6$ $\theta \simeq 1$ rad $\simeq 57°$



Variation 1 : $\mathbf{m}_{\text{ladder}} \neq 0$, $\mu = 0$ at wall.

$\mathbf{f} = \mathbf{N}_2$ $\mathbf{N}_1 = (\mathbf{m}_{\text{person}} + \mathbf{m}_{\text{ladder}})\, \mathbf{g}$ $\mathbf{f} = \mu \mathbf{N}_1$

Rotation condition is

$$\mathbf{m}_{\text{person}}\, \mathbf{g\, s} \cos\theta + \mathbf{m}_{\text{ladder}}\, \mathbf{g} \frac{\mathbf{L}}{2} \cos\theta = \mathbf{N}_2 \mathbf{L} \sin\theta$$

But $\mathbf{N}_2 = \mathbf{f} = \mu\,\mathbf{N}_1 = \mu\,(\mathbf{m}_p + \mathbf{m}_L)\,\mathbf{g}$

$$\mathbf{g}\ \cos\theta \left(\mathbf{m}_p\,\mathbf{s} + \mathbf{m}_L\,\frac{\mathbf{L}}{2}\right) = \mu\,(\mathbf{m}_p + \mathbf{m}_L)\,\mathbf{g}\,\mathbf{L}\ \sin\theta$$

$$\mathbf{m}_p\,\mathbf{s} + \mathbf{m}_L\,\frac{\mathbf{L}}{2} = \mu\,(\mathbf{m}_p + \mathbf{m}_L)\,\mathbf{L}\ \tan\theta$$

$$\mathbf{m}_p\,\frac{\mathbf{s}}{\mathbf{L}} + \frac{\mathbf{m}_L}{2} = \mu\,(\mathbf{m}_p + \mathbf{m}_L)\tan\theta$$

$$\frac{\mathbf{s}}{\mathbf{L}} = \mu\tan\theta\left(1 + \frac{\mathbf{m}_L}{\mathbf{m}_p}\right) - \frac{\mathbf{m}_L}{2\mathbf{m}_p}$$

<u>Variation 2</u> : $\mathbf{m}_{\text{Ladder}} = 0$, $\mu \neq 0$ at wall

$\mathbf{f}_1 = \mathbf{N}_2$ ① $\mathbf{N}_1 = \mathbf{mg} - \mathbf{f}_2$ ② $\mathbf{f}_1 = \mu\,\mathbf{N}_1$ ③ $\mathbf{f}_2 = \mu\,\mathbf{N}_2$ ④

① + ③ $\mathbf{N}_2 = \mu\,\mathbf{N}_1$ ⑤

② + ④ $\mathbf{N}_1 = \mathbf{mg} - \mu\,\mathbf{N}_2$ ⑥

⑤ + ⑥ $\mathbf{N}_1 = \mathbf{mg} - \mu^2\,\mathbf{N}_1$

$$\mathbf{N}_1 = \frac{\mathbf{mg}}{1 + \mu^2}$$

$$\mathbf{N}_2 = \frac{\mu\,\mathbf{mg}}{1 + \mu^2}$$

Torque: $\mathbf{mg\ s}\ \cos\theta = \mathbf{N}_2 \mathbf{L}\ \sin\theta + \mathbf{f}_2 \mathbf{L}\ \cos\theta$

$$= \frac{\mu\, \mathbf{mg}}{1+\mu^2}\ \mathbf{L}\ \sin\theta + \frac{\mu^2\, \mathbf{mg}}{1+\mu^2}\ \mathbf{L}\ \cos\theta$$

$$\mathbf{s} = \frac{\mu}{1+\mu^2}\ \mathbf{L}\ \tan\theta + \frac{\mu^2}{1+\mu^2}\ \mathbf{L}$$

$$\frac{\mathbf{s}}{\mathbf{L}} = \frac{\mu}{1+\mu^2}\left(\tan\theta + \mu\right) = \frac{\mu_1}{1+\mu_1\mu_2}\left(\tan\theta + \mu_2\right)$$

1. Even if $\theta = 0$ $\quad \dfrac{\mathbf{s}}{\mathbf{L}} = \dfrac{\mu^2}{1+\mu^2}$

2. If $\dfrac{\mathbf{s}}{\mathbf{L}} = 1$, $\quad \dfrac{1+\mu^2}{\mu} = \tan\theta + \mu$

$\tan\theta = \dfrac{1}{\mu}$ as before.