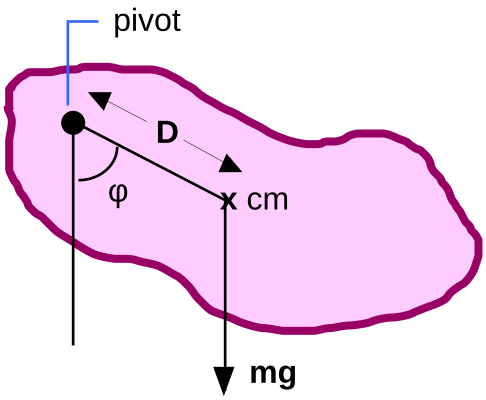
Demonstrations:
•slinky toy to show transverse and longitudinal waves
Text: Fishbane 13-6, 14-1, 14-3
Problems: 51, 54, 59 from Ch. 13; 9, 21, 24 from Ch. 14

What's important:
•physical pendulum
•speed of a wave on a string under tension

Physical pendulum

The simple pendulum of Lecture 30 consisted of a mass attached to the end of a massless string. Consider now the more physical situation in which an arbitrarily-shaped object is hung from a pivot point.



D = distance from pivot to cm position.

The force due to gravity generates a torque τ about the pivot point that tends to restore the body to its equilibrium position.

$$\tau = -(\mathbf{D}\sin\phi)\,\mathbf{mg} \qquad \text{(- sign from clockwise torque)}$$

But we know that

$$\tau = \mathbf{I}\alpha = \mathbf{I}\,(d^2\phi / d\mathbf{t}^2),$$

whence

$$(d^2\phi / d\mathbf{t}^2) = -(\mathbf{mgD} / \mathbf{I})\sin\phi.$$

For small angles ϕ, we can approximate $\sin\phi \sim \phi$, so that the last equation becomes

$$(d^2\phi / d\mathbf{t}^2) = -(\mathbf{mgD} / \mathbf{I})\,\phi.$$

This expression has the same form as the simple harmonic motion expression for springs

$$(d^2\mathbf{x} / d\mathbf{t}^2) = - (\mathbf{k} / \mathbf{m})\, \mathbf{x},$$

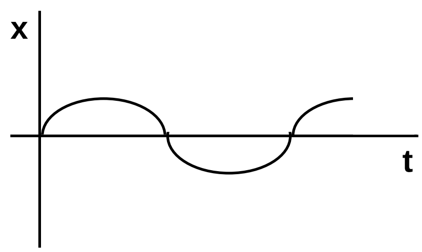
in which $\omega = (\mathbf{k} / \mathbf{m})^{1/2}$. Thus, the physical pendulum executes simple harmonic motion with

$$\omega = (\mathbf{mgD} / \mathbf{I})^{1/2}.$$

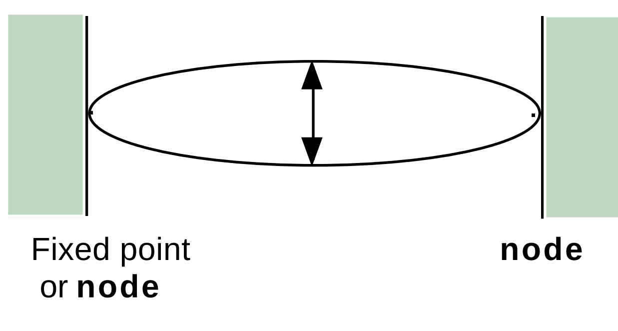
Note: in the simple pendulum, $\mathbf{I} = \mathbf{mD}^2$, and we find $\omega = (\mathbf{g} / \mathbf{D})^{1/2}$ as expected.

Waves on a String

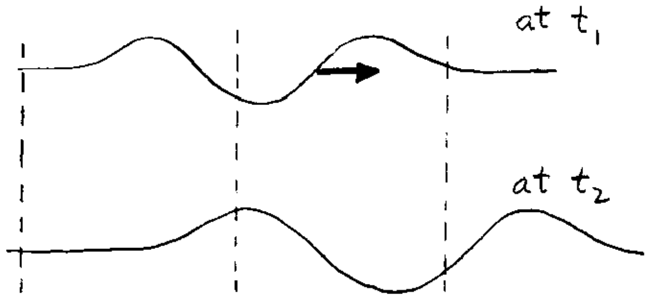
We introduced oscillations in terms of motion of single objects subject to a restoring force. We saw that the resulting motion was wavelike (i.e. a sine or cosine function) as a function of time:



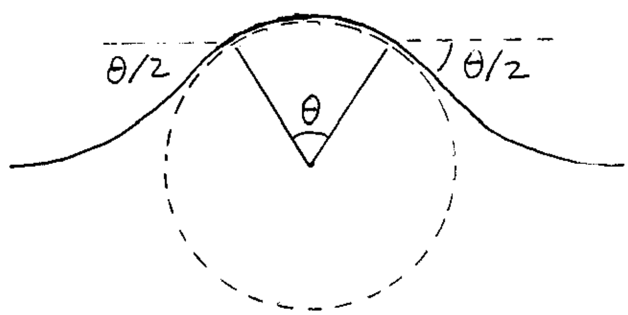
Now let's consider the oscillatory motion of an extended, deformable object, namely a string. We observe both standing waves



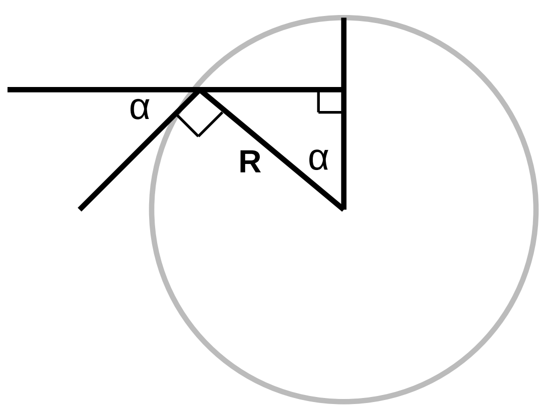
and travelling waves (time $\mathbf{t}_2$ is greater than $\mathbf{t}_1$)



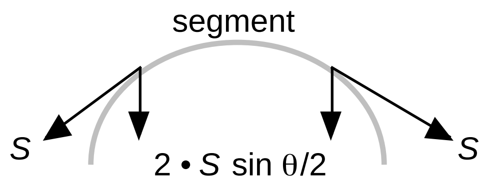
The frequency of a wave depends upon its velocity. We can find the form of the relationship by considering a wave pulse. Approximate the segement at the top of the wave as part of a circle with arc length $\Delta\mathbf{s} = \mathbf{R}\theta$, as in the diagram



where the relationships among the angles is obtained from simple trigonometry



The forces on the segment $\Delta\ell$ are from the tension S at each end of the segment



where S is used to represent tension, rather than the more obvious T, since T is already used for the oscillation period. The components of S acting to restore the segment to its equilibrium position are $S \sin\theta/2$ at each end. Suppose that θ is small, so that we can make the small angle approximation $\sin\theta / 2 \sim \theta/2$. The net force that the segment experiences in the transverse direction is then

$$2\, S \sin \theta/2 \sim S\theta \qquad (1)$$

Now, θ is related to the arc length $\Delta\ell$ by

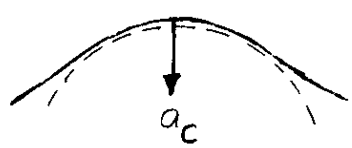
$$\theta = \Delta\ell / R$$

so that the expression for the restoring force (1) can be rewritten as

$$S\ \Delta\ell / R.$$

This force gives the string element an acceleration in the transverse direction, which can be thought of as follows. Imagine that we observe the wave by travelling along beside it with a velocity v. If we look at the segment at the top of the wave, then it appears to experience a centripetal acceleration a_c, given by the usual expression

$$a_c = v^2 / R.$$



Thus, the force $S\ \Delta\ell / R$ gives the segment of mass $\mu\ \Delta\ell$ (where μ is the mass per unit length of string) an acceleration v^2 / R. Substituting into Newton's Second Law

$$S = ma,$$

we have

$$S \, \Delta\ell / R = (\mu \, \Delta\ell)(v^2 / R)$$

$$\Rightarrow \quad S = \mu v^2$$

or

$$v = (S / \mu)^{1/2} \qquad (2)$$

Equation (2) gives the velocity of the wave in terms of the tension in the string.

Note:
(i) both R and θ have disappeared from Eq. (2).
(ii) the units work out as expected:

$[force] \sim \text{kg m}^2 / \text{s}^2$

$[\mu] \sim \text{kg / m}$

$\Rightarrow \ [\,(force / \mu)^{1/2}] \sim (\text{m}^2 / \text{s}^2)^{1/2} = \text{m / s}.$