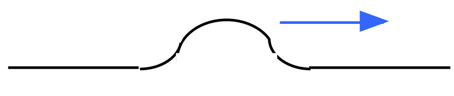
Demonstrations:
•retort stand, elastic band, 0.5 kg mass
Text: Fishbane 14-2, 14-5
Problems: 8, 11, 31, 34 from Ch. 14

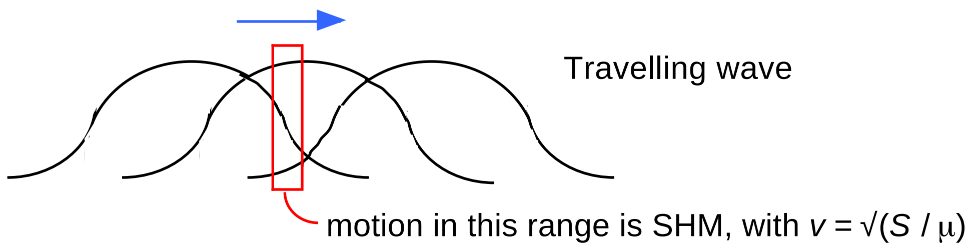
What's important:
•vibrational frequency as a function of tension, mass
•power of a wave

Standing and Travelling Waves

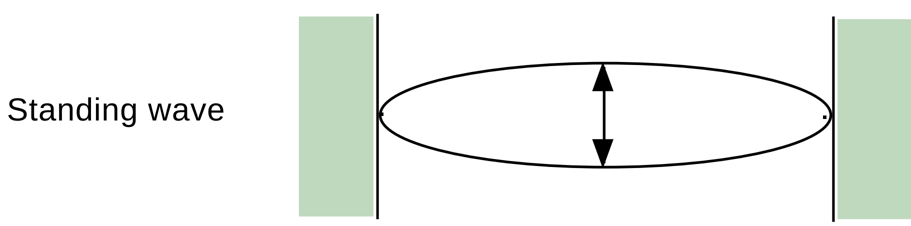
If we pluck a long string and release it, we can see a travelling wave move off from the place where the wave was plucked.



At any given point on the string, the movement of the string transverse to the direction of motion is SHM:



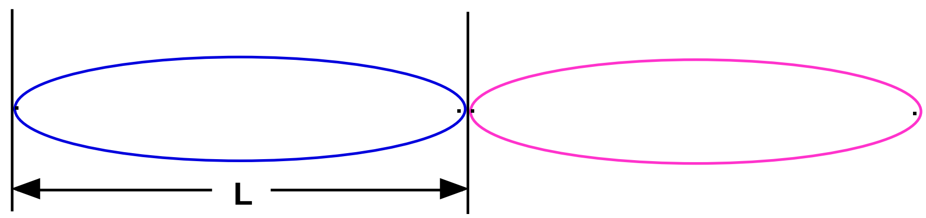
Now, if the string is fixed at each end, then there is no motion at the end points, and the wave is called a *standing wave*:



Any element of a standing wave executes SHM corresponding to a velocity of $(S/\mu)^{1/2}$, even though the wave does not appear to move along the string. Because the wave velocity may not be apparent, it is sometimes more intuitive to think in terms of f and λ [frequency and wavelength].

---> $v = f\lambda$ ---> $f = v / \lambda = (S/\mu)^{1/2} / \lambda$.

The wavelength of the longest standing wave allowed by fixed walls separated by a length **L** is



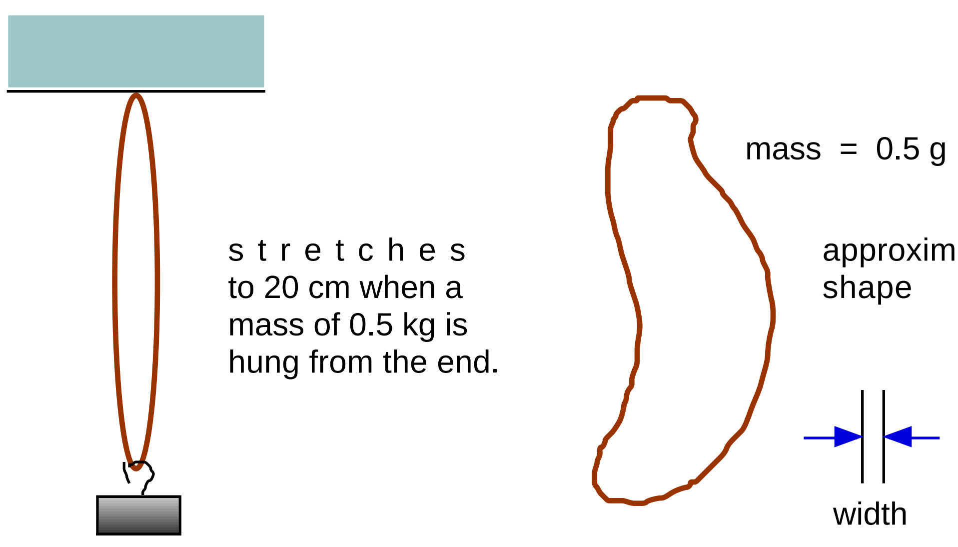
$$\lambda = 2\mathbf{L}$$

Hence, the *lowest* frequency (largest λ) of a standing wave is

$$f = (S/\mu)^{1/2} / 2L$$

Demonstration

Hang a mass from an elastic band and pluck it. The following results were obtained with an "average" elastic band



For *one* side (of a two "sided" elastic)

$$\mu = \text{mass per unit length} = \frac{0.5 \times 10^{-3} / 2}{0.20}$$

0.5 g for total band

20 cm

$$\simeq 10^{-3} \text{ kg / m}$$

$$S = \frac{0.5 \times 9.8}{2}$$

0.5 kg mass is supported by two sides

$$= 2.5 \text{ N}$$

$$v = \sqrt{\frac{S}{\mu}} = \sqrt{\frac{2.5}{10^{-3}}} = \sqrt{2500} = 50 \text{ m / s}$$

The corresponding frequency is $\mathbf{f} = \frac{\mathbf{v}}{\lambda} = \frac{\mathbf{v}}{2\mathbf{L}} = \frac{50}{2 \times 0.2} = 125 \text{ sec}^{-1}$

This compares to middle C at 256 Hz.

Power of a Wave

When we looked at the energy of an oscillating spring, we found that the total energy was

$$E = (1/2)\, kA^2 = \text{potential energy at maximum extension.}$$

This expression can be rewritten as

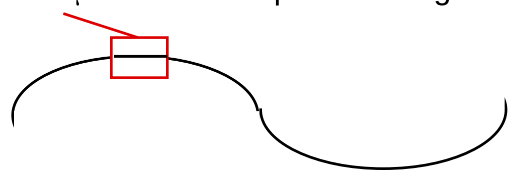
$$E = (1/2)\, m\, \omega^2 A^2 \qquad (1)$$

by using

$$k = m\,\omega^2.$$

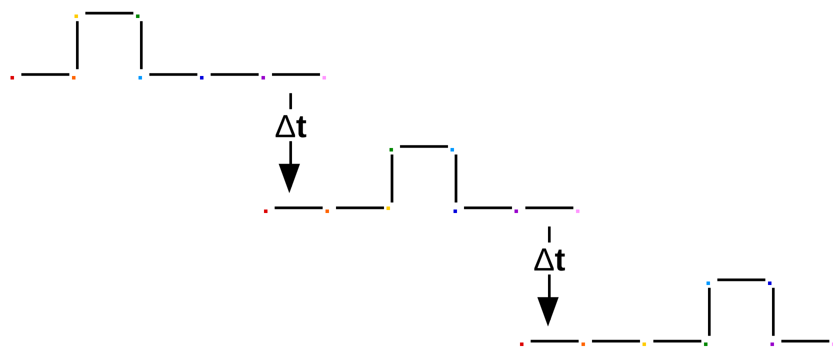
Although derived for a spring, Eq. (1) is valid for any oscillating system. For a wave on a string, we can use $E = 1/2\ \Delta m\, \omega^2 A^2$ for each element of mass Δm and length Δx on the string:

$\Delta m = \mu \Delta x$ where Δx is the length of the element and μ is the mass per unit length.



$$\Delta E = 1/2 \ \mu \ \Delta x \ \omega^2 A^2$$

Now, consider a wave pulse where the energy is transmitted from one element Δx to the next in a time Δt



In each unit of time, the energy ΔE has moved a distance Δx down the string. the power of the wave (energy per unit time) is

$$P = \frac{\Delta E}{\Delta t} = \frac{1/2 \ \mu \ \omega^2 A^2 \ \Delta x}{\Delta t} = 1/2 \ \mu \ \omega^2 A^2 \ v$$

Note: power goes like

ω^2 high frequency more power

A^2 amplitude <u>squared</u>