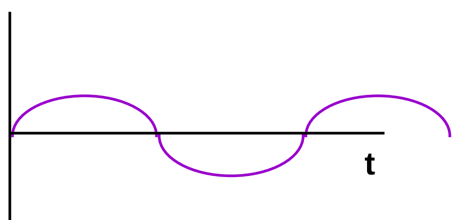
Demonstrations:
•interference, ripple tank
Text: Fishbane 15-1, 15-2

What's Important:
•travelling waves and phase angle
•constructive and destructive interference

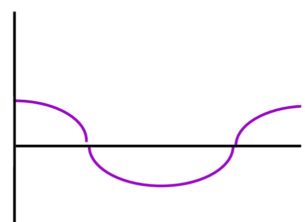
Travelling waves

When we solved the equations for simple harmonic motion, we said that either a sine or a cosine function was a valid solution. One chooses the solution according to the desired value of the displacement **y** at **t** = 0. (Note: we have changed notation and used **y** as displacement, for reasons that become obvious in the next paragraph).

y(**t**) = displacement



$y = A \sin \omega t$



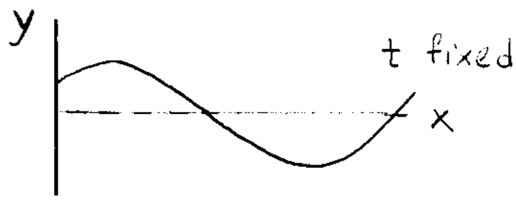
$y = A \cos \omega t$

These solutions have |**y**(**t**=0)| = 0 or **A**. A more general solution that allows for the complete range of **y**(**t**=0) is

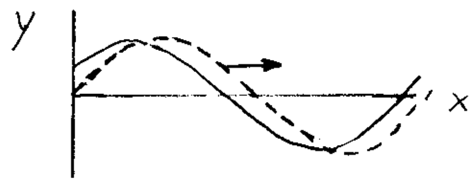
$$y(t) = A \sin (\omega t + \delta)$$

where δ is called the phase angle. The presence of δ allows the wave pattern along the time-axis to be shifted.

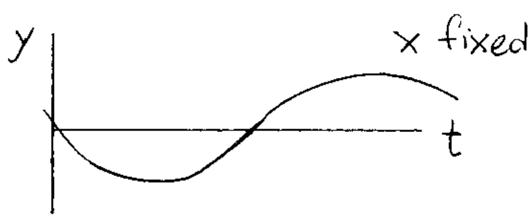
When we have a travelling wave, as opposed to a spring oscillating in one dimension), the displacement **y** depends on both time and the position in space **x** at which the displacement is measured. At a fixed time t_1, the displacement as a function of **x** is denoted by $y(x,t_1)$, and has the graphical representation:



At a later time $\mathbf{t}_2$, the wave has moved to the right, and the displacement $\mathbf{y}(\mathbf{x},\mathbf{t}_2)$ looks like



Similarly, for a fixed position ($\mathbf{x} = \mathbf{x}_1$ fixed) the displacement as a function of time is $\mathbf{y}(\mathbf{x}_1,\mathbf{t})$ looks like



Let's write out mathematically what is contained in the diagrams. We need an expression for the displacement from equilibrium as a function of **x**, **t**:

$\mathbf{y}\,(\mathbf{x}, \mathbf{t})$ = displacement from equilibrium at position **x** and time **t**:

$$= \mathbf{A} \sin \left[2\pi \left(\frac{\mathbf{x}}{\lambda} - \frac{\mathbf{t}}{\mathbf{T}} \right) \right]$$ travels to right

λ = wavelength $\mathbf{T}$ = period of oscillation

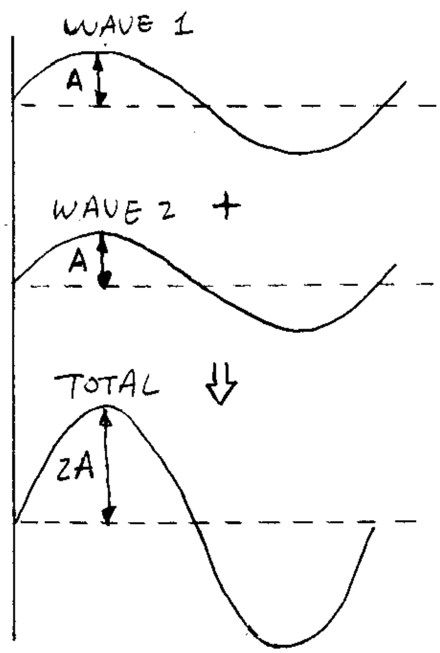
a wave travelling to the left has a very similar functional form, but with the opposite sign for the "time" argument:

$$\mathbf{y}\,(\mathbf{x}, \mathbf{t}) \;=\; \mathbf{A}\;\sin\left[2\pi\left(\frac{\mathbf{x}}{\lambda} + \frac{\mathbf{t}}{\mathbf{T}}\right)\right]$$ travels to left

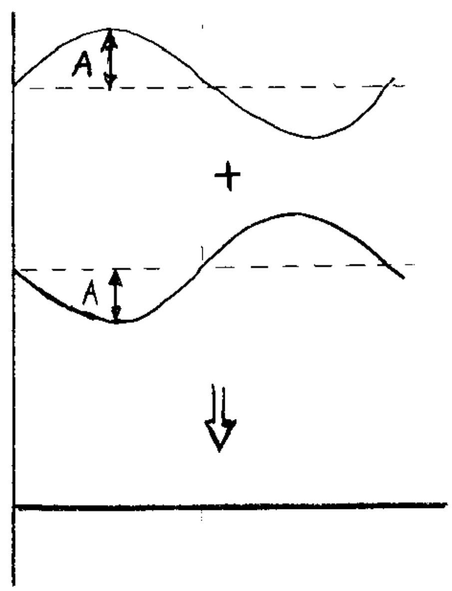
Further, one could add a further phase angle to the argument.

Superposition principle

What happens when two waves are travelling along the same medium? The waves can have arbitrary phase angle with respect to one another. The displacement of the medium from equilibrium, **y**(**x**,**t**), at a given value of **x** and **t** is the algebraic sum of the individual displacements of the waves. Consider the situation shown in the figure below, where the combined wave is displayed at a fixed time. That is, the figure shows **y**(**x**, fixed **t**); of course, the drawings also could represent **y** as a function of **t** at fixed **x**.



Constructive interference



Destructive interference

The displacement of the combined wave is just the algebraic sum (watch for the signs) of the indiviudal displacements,

$$\mathbf{y}_{total}\ (\mathbf{x}, \mathbf{t}) = \mathbf{y}_1\ (\mathbf{x}, \mathbf{t}) + \mathbf{y}_2\ (\mathbf{x}, \mathbf{t})$$

This is refered to as the superposition principle.

Summary of waves and oscillations

1. Basic definitions: $\mathbf{T} = 1/\mathbf{f}$ $\quad\omega = 2\pi\mathbf{f}$

2. Speed: $\mathbf{c} = \lambda\mathbf{f}$

3. Functions: $\mathbf{x}(\mathbf{t}) = \mathbf{A}\ \cos \omega\mathbf{t}$ where $\mathbf{A}$ = amplitude

   or $\mathbf{A}\ \sin(\omega\mathbf{t} + \delta)$ in general

4. Frequency of SHM spring: $\omega = \sqrt{\frac{\mathbf{k}}{\mathbf{m}}}$ pendulum: $\omega = \sqrt{\frac{\mathbf{g}}{\mathbf{l}}}$

5. Energy: $\mathbf{K} = 1/2\ \mathbf{mv}^2$ $\quad\mathbf{U} = 1/2\ \mathbf{kx}^2$

   $\mathbf{K} + \mathbf{U} = 1/2\ \mathbf{kA}^2 = \text{constant}$

6. Doppler Shift $\frac{\Delta\lambda}{\lambda_0} = \pm \frac{\mathbf{v}}{\mathbf{c}}$ $\quad\Delta\lambda = \lambda' - \lambda_0$

   \+ if away
   \- if towards

7. Waves on a string $\mathbf{v} = (\mathbf{S} / \mu)^{1/2}$

   power = $\mathbf{P} = (1/2)\ \mu\ \mathbf{A}^2\ \omega^2\ \mathbf{v}$

8. Travelling waves

$$\mathbf{y}\ (\mathbf{x}, \mathbf{t}) = \mathbf{A}\ \sin\left[2\pi\left(\frac{\mathbf{x}}{\lambda} +/- \frac{\mathbf{t}}{\mathbf{T}}\right)\right]$$ (+) to left (-) to right

9. Superposition of waves $\mathbf{y}_{total}\ (\mathbf{x},\mathbf{t}) = \Sigma_i\ \mathbf{y}_i\ (\mathbf{x},\mathbf{t})$